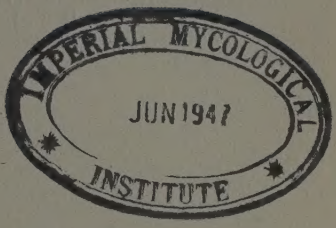
COMPTES RENDUS
(DOKLADY)
DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1944



VOLUME XLIV

№ 9

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

AVIS AUX AUTEURS

Les «Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS» paraissent tous les dix jours. Neuf numéros composent un volume pourvu d'une table des matières. Il y a quatre volumes par année.

Les «Comptes Rendus de l'Académie des Sciences de l'URSS» en de courtes communications précisent l'essentiel des travaux en préparation. Ces articles, ne dépassant pas en règle générale quatre pages, sont donnés en anglais, en français ou en allemand. Ils embrassent les sciences mathématiques, physiques, naturelles et appliquées, mettant au jour les résultats de recherches scientifiques en cours ou qui viennent d'être achevées, et représentant ainsi les plus récentes données de l'investigation scientifique.

L'auteur a droit à 50 exemplaires de son article.

Adressez les manuscrits à la Rédaction des «Comptes Rendus», Moscou, Wolkhonka, 14.

30 Septembre

COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

COMITÉ DE RÉDACTION

A. Arbusov, de l'Académie, **V. Chlopin**, de l'Académie, **A. Deborin**, de l'Académie, **A. Fersman**, de l'Académie, **A. Frumkin**, de l'Académie, **A. Kolmogoroff**, de l'Académie (vice rédacteur), **L. Orbeli**, de l'Académie (rédacteur en chef), **A. Richter**, de l'Académie, **S. Soboleff**, de l'Académie, **S. Wawilow**, de l'Académie (vice rédacteur)

NOUVELLE SÉRIE

12-me année

Paraissant tous les dix jours

1944

VOLUME XLIV, № 9

TABLE DES MATIÈRES

Pages

MATHÉMATIQUES

J. Geronimus. On Some Distribution Functions Connected with Systems of Polynomials 355

L. J. Neushuler. On Tabulation of Systems of Non-explicit Functions of Two Variables 360

PHYSIQUE

V. I. Veksler. On a New Method of Acceleration of Relativistic Particles . . 365

ÉLECTRICITÉ

P. L. Kalantarov and **L. A. Zeitlin.** Determination of Resonance Curve Maxima for a Symmetric Homogeneous Recurrent Circuit 369

CHIMIE

G. V. Pigulevsky and **A. T. Ryskalchuk.** Raman Effect Investigation of Turpentine from *Pinus silvestris* 372

I. J. Postovsky, N. P. Bednyaghina and **M. A. Mikhailova.** On the Sulphur Compounds in Petroleums 375

CHIMIE DU GLOBE

B. N. Melentiev. Formulae for Determining Nepheline and Pyroxenes in Apatite-Nepheline Ores of the Chibiny Tundras 378

GÉOLOGIE

H. M. Abdullaev. The Province of Scheelite-bearing Skarns in Central Asia . 381
K. I. Makov. On the Formation of Underground Waters of the Dnieper-Donetz Depression 386

CYTOLOGIE

G. Roskin and G. Kharlova. Zymonucleic Acid in Cells of a Normal Regenerate and in those of Malignant Growth 389

PHYSIOLOGIE VÉGÉTALE

O. J. Sobolevskaya. Rubber Formation in Roots of Kok-saghyz Dug under Conditions of Impeded Gas Exchange 392

PHYSIOLOGIE

V. Schroeuder. Les spermatoxines et le problème de la régulation artificielle du sexe chez les mammifères 396

Le présent numéro est suivi de l'indice au volume XLIV.

Édition de l'Académie des Sciences de l'URSS
Moscou 1944

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 9

MATHEMATICS

ON SOME DISTRIBUTION FUNCTIONS CONNECTED WITH SYSTEMS OF POLYNOMIALS

By J. GERONIMUS

(Communicated by S. N. Bernstein, Member of the Academy, 28. II. 1944)

I. Consider a system of polynomials $\{P_n(z)\}_0^\infty$

$$zP_n(z)=\sum_{k=0}^{n+1}\alpha_{nk}P_k(z);\ P_n(z)=\prod_{i=1}^{n}(z-\xi_i^{(n)}),\ \xi_1^{(n)}\leqslant\xi_2^{(n)}\leqslant\ldots\leqslant\xi_n^{(n)} \tag{1}$$

all their roots lie on the segment $<-1,\ +1>$; introduce the matrix

$$A_n=\|\alpha_{ij}\|_0^{n-1};\ \alpha_{i,i+1}=1;\ \alpha_{ij}=0;\ j>i+1 \tag{2}$$

and consider its powers

$$A_n^k=\|\alpha_{ij}^{(k,n)}\|_0^{n-1},\ (k=1,\ 2,\ \ldots);\ \alpha_{ij}^{(1,n)}=\alpha_{ij} \tag{2'}$$

Introduce the step-functions $\{\psi_n(x)\}_1^\infty$ where the function $\psi_n(x)$ has the positive jump $\frac{k}{n}$ at each root of multiplicity k of the polynomial $P_n(z)$; we have further

$$\psi_n(x)\equiv 0,\ -\infty<x<\xi_1^{(n)};\ \psi_n(x)\equiv 1,\ \xi_n^{(n)}<x<+\infty$$

If the sequence $\{\psi_n(x)\}$ converges substantially (*i. e.* on a dense set) to some limit function $\psi(x)$, we shall call $\psi(x)$ the distribution function of zeros of the system $\{P_n(z)\}_1^\infty$ on the segment $<-1,\ +1>$.

Consider now the Pólya quadrature process [1]; to each function $f(x)$, continuous on the segment $<-1,\ +1>$, there corresponds the functional

$$Q_n(f)=\sum_{k=1}^{n}\lambda_k^{(n)}f(\xi_k^{(n)}) \tag{3}$$

$\{\lambda_k^{(n)}\}_1^n$ being arbitrary numbers; in the case where

$$n\lambda_k^{(n)}=1+o(1)\ (k=1,\ 2,\ \ldots,\ n) \tag{4}$$

we shall speak of Pólya-Tchebycheff quadrature process.

Theorem 1. *The existence for all k of the finite limits*

$$\lim_{n\to\infty}\left\{\frac{1}{n}\sum_{i=0}^{n-1}\alpha_{ii}^{(k,n)}\right\} \tag{5}$$

is equivalent to the existence of the distribution function of zeros $\psi(x)$ *and is equivalent to the convergence of Pólya-Tchebycheff quadrature process, i. e. to the existence of the limit of the functional*

$$\lim_{n\to\infty} Q_n(f) = \lim_{n\to\infty} \sum_{k=1}^{n} \lambda_k^{(n)} f(\xi_k^{(n)}) = \int_{-1}^{1} f(x)\, d\psi(x) \tag{6}$$

To prove this we establish the equality

$$\sum_{i=1}^{n} (\xi_i^{(n)})^k = \sum_{i=0}^{n-1} \alpha_{ii}^{(k,n)}, \quad (k = 1, 2, \ldots) \tag{7}$$

and apply the theorem of Fréchet-Shohat (2) and Helly.

Hence follows the existence of the limits

$$\lim_{n\to\infty} \left\{ \frac{1}{n} \sum_{k=1}^{n} f(\xi_k^{(n)}) \right\} = \lim_{n\to\infty} \int_{-1}^{1} f(x)\, d\psi_n(x) = \int_{-1}^{1} f(x)\, d\psi(x) \tag{8}$$

and, in particular,

$$\lim_{n\to\infty} \frac{P_n'(z)}{nP_n(z)} = \int_{-1}^{1} \frac{d\psi(x)}{z-x}; \quad \lim_{n\to\infty} \sqrt[n]{P_n(z)} =$$

$$= \exp\left\{ \int_{-1}^{1} \lg(z-x)\, d\psi(x) \right\}, \quad z \overline{\in} <-1, +1> \tag{9}$$

II. Denote by E the set of all zeros of the polynomials $\{P_n(z)\}_1^\infty$; by $\overline{E}$, its closure; and by $d(\overline{E})$, its transfinite diameter (or capacity) supposed positive; denote by $\mu(x)$ the Robin distribution function, *i. e.* such non-negative distribution of the unit-mass on $\overline{E}$ by which the logarithmic potential of this distribution is constant on $\overline{E}$, *i. e.*

$$\int_{-1}^{1} \lg \frac{1}{|z-x|}\, d\mu(x) = \gamma, \quad z \in \overline{E} \tag{10}$$

Theorem 2. *For the equivalence of the functions* $\psi(x)$ *and* $\mu(x)$ *is necessary and sufficient the existence of the limit*

$$\lim_{n\to\infty} \sqrt[n]{M_n} = \lim_{n\to\infty} \sqrt[n]{\mathcal{E}_n} = d(\overline{E}) = e^{-\gamma} \tag{11}$$

M_n *being the maximal modulus of* $P_n(z)$ *on* $\overline{E}$; $\mathcal{E}_n$ *is the minimal deviation from zero on* $\overline{E}$ *of the polynomial* $z^n + \ldots$

If, in particular, $\overline{E}$ reduces to the segment $<-1, +1>$, then the condition

$$\lim_{n\to\infty} \sqrt[n]{M_n} = \frac{1}{2} \tag{11'}$$

is both necessary and sufficient for the equality

$$\psi(x) = \mu(x) = 1 - \frac{1}{\pi} \arccos x$$

To prove the Theorem 2 we consider the logarithmic potential

$$u_n(z)=\frac{1}{n}\lg\frac{1}{|P_n(z)|}=\int_{-1}^{1}\lg\frac{1}{|z-x|}\,d\psi_n(x) \tag{12}$$

where we have for $z\in\overline{E}$ $u_n(z)\geqslant\frac{1}{n}\lg\frac{1}{M_n}$; consider now the harmonic function

$$v_n(z)=u_n(z)-\frac{1}{n}\lg\frac{1}{M_n}+g(z,\infty) \tag{13}$$

$g(z,\infty)$ being the Green function of the domain G complementary to the set $\overline{E}$. On the boundary of G we have $v_n(z)\geqslant 0$; consequently, $v_n(z)\geqslant 0$ everywhere in G and, in particular, $v_n(\infty)=\gamma-\frac{1}{n}\lg\frac{1}{M_n}\geqslant 0$. Since by condition

$$\lim_{n\to\infty}\left\{\frac{1}{n}\lg\frac{1}{M_n}\right\}=\gamma$$

we have in the domain G [3]

$$\lim_{n\to\infty}v_n(z)=0;\quad\lim_{n\to\infty}u_n(z)=\lim_{n\to\infty}\int_{-1}^{1}\lg\frac{1}{|z-x|}\,d\psi_n(x)=\int_{-1}^{1}\lg\frac{1}{|z-x|}\,d\mu(x)$$

whence follows the equivalence of the functions $\psi(x)$ and $\mu(x)$; in the same way we prove the necessity of the condition (11).

III. Let now our polynomials $\{P_n(z)\}_0^\infty$ be orthogonal on the segment $<-1,+1>$ with regard to some distribution $d\sigma(x)$, $\sigma(x)$ being a bounded non-decreasing function with infinitely many points of increase on $<-1,+1>$ with $\int_{-1}^{1}d\sigma(x)=1$; in this case A_n is a Jacobi matrix

$$\alpha_{ii}=a_{i+1};\quad\alpha_{i,i-1}=\lambda_{i+1}>0;\quad\alpha_{ij}=0,\ |i-j|>1.$$

Theorem 3. *The condition of the equivalence of the functions $\psi(x)$ and $\sigma(x)$ is equivalent to the condition*

$$\lim_{n\to\infty}\left\{\frac{1}{n}\sum_{i=n}^{n-1}\alpha_{ii}^{(k,n)}\right\}=\alpha_{00}^{(k,n)*} \tag{14}$$

and is equivalent to the condition of the coincidence of Gauss-Christoffel quadrature process (for $n\to\infty$)

$$\sum_{k=1}^{n}g_k^{(n)}(\xi_k^{(n)})^r=\int_{-1}^{1}x^r\,d\sigma(x),\quad r\leqslant 2n-1 \tag{15}$$

with that of Pólya-Tchebycheff [3,4] i. e. to the existence of the limits for the Christoffel numbers

$$\lim_{n\to\infty}ng_k^{(n)}=1,\quad(k=1,2,\ldots n). \tag{16}$$

For an orthogonal system we can give in some particular cases a very simple condition sufficient for the equivalence of the functions $\psi(x)$ and $\mu(x)$,

---

* The element α_{00}^{kn} is the same for all $n\geqslant\frac{k+1}{2}$.

Let E_0 be a set of segments $\{e_k\}_1^m$ of length $\geqslant h$, lying on the segment $<-1, +1>$; let the set E of points of increase of the function $\sigma(x)$ be a sum of two sets $E=E_1+E_2$, where $\overline{E}_1=E_0$, E_2 being a denumerable isolated set whose limit points belong to E_0.

Introduce the «modulus of increase» of the function $\sigma(x)$ on E_0 [4]

$$a(\delta)=\inf\{\sigma(x+\delta)-\sigma(x)\}, \quad x, x+\delta\in e_k, \quad (k=1, 2, \ldots, m). \tag{17}$$

Theorem 4. *If the modulus of increase $a(\delta)$ of the function $\sigma(x)$ is subject to the condition*

$$\lim_{\delta\to 0}\{\sqrt{\delta}\lg a(\delta)\}=0 \tag{18}$$

then there exists the limit

$$\lim_{n\to\infty}\sqrt[n]{M_n}=d(E_0)=d(\overline{E}); \quad |P_n(x)|\leqslant M_n, \quad x\in E_0 \tag{19}$$

According to the Theorem 2, hence follows the equivalence of the functions $\psi(x)$ and $\mu(x)$.

In particular, the condition (18) always holds true if $d\sigma(x)=p(x)\,dx$, where $p(x)\geqslant p_0>0$ on E_0; in this case $a(\delta)=p_0\delta$. The proof of the Theorem 4 follows from the double inequality [4,5]

$$C^2M_n^2a(\delta_n)\leqslant h_n=\frac{1}{2}\int_{-1}^{1}P_n^2(x)\,d\sigma(x)\leqslant\mathcal{E}_n\leqslant M_n^2 \tag{20}$$

C being a constant, $\delta_n=A/n^2$; to prove this we apply (after D. Jackson) A. Markoff's estimate for $P_n'(z)$ on e_k.

IV. Consider now the polynomials $\{P_n(z)\}_0^\infty$ orthogonal with regard to the distribution

$$d\sigma(x)=(ax+b)\,d\mu(x); \quad a\neq 0; \quad ax+b\geqslant 0, \quad x\in<-1,+1> \tag{21}$$

Introduce the polynomials $\{R_{n-1}(z)\}_1^\infty$ and the functions $\{Q_n(z)\}_0^\infty$ of the second kind

$$R_{n-1}(z)=\int_{-1}^{1}\frac{P_n(z)-P_n(x)}{z-x}\,d\sigma(x); \quad Q_n(z)=\int_{-1}^{1}\frac{P_n(x)\,d\sigma(x)}{z-x}; \tag{22}$$

they are connected by the relation

$$Q_0(z)P_n(z)=R_{n-1}(z)+Q_n(z), \quad (n=0, 1, 2, \ldots) \tag{23}$$

We shall denote by asterisk the Cauchy principal value of the integral

$$Q_n^*(z)=\int_{-1}^{1}\frac{P_n(x)\,d\sigma(x)}{z-x}, \quad (n=0, 1, 2, \ldots); \quad z\in E \tag{24}$$

Theorem 5. *Under condition* (21) *the functions* $\left\{-\frac{1}{a}Q_n^*(z)\right\}_0^\infty$ *form a system of polynomials* $\{\tilde{P}_n(z)\}_0^\infty$ *orthogonal on the segment* $<-1, +1>$ *with regard to some distribution* $d\tilde{\sigma}(x)$; *we have*

$$\left\{a+\int_{-1}^{1}\frac{d\sigma(x)}{z-x}\right\}\left\{a-\int_{-1}^{1}\frac{d\tilde{\sigma}(x)}{z-x}\right\}=a^2, \quad z\in<-1, +1> \tag{25}$$

at the points where the function $\sigma(x)$ *has the derivative* $p(x)>0$, *the function* $\tilde{\sigma}(x)$ *has the derivative* $\tilde{p}(x)$ and

$$p(x)\,\tilde{p}(x)=a^2 \tag{26}$$

*We have further the relations**

$$\int_{-1}^{+1}\frac{P_n(x)\,d\sigma(x)}{z-x}=\tilde{P}_n(x); \qquad \int_{-1}^{+1}\frac{\tilde{P}_n(x)\,d\tilde{\sigma}(x)}{z-x}=P_n(z), \quad z\in E, \quad (n=0,1,\ldots) \tag{27}$$

The polynomials of the second kind are the same for both systems

$$\tilde{R}_{n-1}(z)=R_{n-1}(z), \quad (n=1,2,\ldots) \tag{28}$$

The proof follows from the fact that the condition (10) implies the condition

$$\int_{-1}^{+1}\frac{d\sigma(x)}{z-x}=0, \quad z\in E \tag{29}$$

Owing to (23) it is clear that the functions $Q_n^*(z)$ are the polynomials of corresponding degrees and that the polynomials $\{\tilde{P}_n(z)\}_0^\infty$ satisfy to the same finite difference equation

$$y_n=(z-\alpha_n)\,y_{n-1}-\lambda_n\,y_{n-2}, \quad (n^*=1,2,\ldots), \quad y_{-1}=0 \tag{30}$$

as the polynomials $\{P_n(z)\}_0^\infty$; we have, moreover,

$$P_0=\tilde{P}_0=1; \qquad P_1(z)=z-\alpha_1; \qquad \tilde{P}_1(z)=z-\tilde{\alpha}_1; \qquad \tilde{\alpha}_1=\alpha_1-\frac{1}{a} \tag{31}$$

Hence using the continued fractions

$$\int_{-1}^{1}\frac{d\sigma(x)}{z-x}=\frac{1}{|z-\alpha_1|}-\frac{\lambda_2}{|z-\alpha_2|}-\ldots,$$

$$z\,\bar{\in}<-1,+1> \tag{32}$$

$$\int_{-1}^{1}\frac{d\tilde{\sigma}(x)}{z-x}=\frac{1}{\left|z-\alpha_1+\frac{1}{2}\right|}-\frac{\lambda_2}{|z-\alpha_2|}-\ldots.$$

we deduce (25) and (28); the formula (26) follows at once from the Stieltjes-Perron inversion formula.

Received
28. II. 1944.

REFERENCES

[1] G. Pólya, Math. ZS., 37, 264 (1933). [2] M. Fréchet and J. Shohat, Trans. Amer. Math. Soc., 33, 533 (1931). [3] R. Nevanlinna, Eindeutige analytische Funktionen, 1941. [4] J. Shohat, Annali di Mat., XVIII, 201 (1939). [5] J. Shohat, Amer. Journ. of Math., LV, 218 (1933).

* Since $\tilde{h}_n=h_n (n=0,1,2,\ldots)$ we normalize both orthogonal systems at the same time by division by $\{\sqrt{h_n}\}_0^\infty$. The orthogonal systems of polynomials, satisfying to the conditions (25—29), have been considered for the first time by N. Achyeser.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 9

MATHEMATICS

ON TABULATION OF SYSTEMS OF NON-EXPLICIT FUNCTIONS OF TWO VARIABLES

By L. J. NEUSHULER

(Communicated by S. L. Soboleff, Member of the Academy, 19. I. 1944)

We are to consider a system made up by pairs of non-explicit functions of two variables, x_i and x_j, defined by the equation

$$f_i(u, v, x_i) = 0; \quad f_j(u, v, x_j) = 0 \qquad \text{(A)}$$

where i and j are values of a parameter p.

The tabular constructions described in the present paper are intended to solve the system (A) for a small as well large number of pairs of values of the parameter p, including the case where the approximate values of u and v are known beforehand. To make full use of interpolation as a means for condensing the volume of the table is here a more urgent problem than when one function of two variables is tabulated. It is expedient, if only space permits it, to give the tabular values with respect to one variable x_j at such intervals as entail no necessity of interpolation, and to give the tabular values with respect to the other variable x_i at the longest possible intervals, leaving room for interpolation.

It is with the purpose of simplifying the interpolation technique that the author arrived at constructions consisting (for every system of values i and j) in a series of tables whose head-line has one of the following forms:

$x_i = \overline{x_i}$

u	x_j	v	Δx_i	Δu	Δx_i	Δv	u	x_j	v	Δx_i	Δu	Δx_i	Δv	... (a)

$x_i = \overline{x_i}$

Δx_i						
for Δu	for Δv	u	x_j	v	Δu	Δv	u	x_j	v	Δu	Δv	... (b)
---	---	---	---	---	---	---	---	---	---	---	---	---

$x_i = \overline{x_i}$

Δu	Δv	u	x_j	v	Δx_i	Δx_i	u	x_j	v	Δx_i	Δx_i	... (c)

The arrangement and use of such tables will be discussed presently by taking one of the above forms, say, (c), as an example. Each of the tables (c) corresponds to some definite value $x_i = \overline{x_i}$. The intervals between the successive values $\overline{x_i}$ have their length determined by the requisite accuracy of the

tabular results and by the number of lines which a page of the given book-size can hold. The values of x_j are given against the tabulated x_i in succession for each necessary value of u with requisite accuracy.

The procedure in using the table (c) is as follows: in the table marked by the given value of x_i we find the corresponding value of x_j. The values of u and v in the same line to the left and right of the found value of x_j are the sought values.

If we find no table marked by the given value of x_i, then:

1) We look in the table marked by the nearest value of x_i for the values of u and v corresponding to the tabulated value of x_i.

2) In the next column to the right of that in which the value of x_j has been found, we find in an n-th line the difference between the given value of x_i and the tabulated one (or the next greater difference).

The value of Δu that we find in the same line n in the next column to the left is just the correction for u.

3) In the following Δx_i column we again find the difference between the given and the tabulated values of x_i. The sought correction for v will be found in the same line under Δv. The rule of finding the sign of the correction is determined by the given system (A) and is indicated, accordingly, over the columns Δu and Δv.

Note. A particularly convenient construction can be suggested if the ratio $\Delta v/\Delta u$ differs but little from unity.

$x_i = \overline{x_i}$

u	x_j	v	Δx_i	Δu	Δv	u	x_j	v	Δx_i	Δu	Δv

...(d)

In this case Δx_i is looked up in one column, and the corrections Δu and Δv are always found by its side.

In practice it is sometimes advisable to eliminate all arithmetic operations. In such cases there can be recommended a table consisting for every system of values i, j of a series of tables having the following form:

	$\Delta u =$	$d_1 d_2 d_3 \cdots$
	x_j	Δx_i
$x_i = \overline{x_i}$	c_0 c_1 . . . c_l	II

(e)

u	$a_1 a_2 \cdots a_q$ $x_j x_j \cdots x_j$	u
b_1 b_2 . . . b_m	I	III

Here Part I gives the basic values of u, *i. e.* those corresponding to the tabulated values of x_i and to the given value of x_j. The values of u are formed

by suffixing to the sets of figures forming the numbers $a_1, a_2, \ldots$ (the higher place figures of u) the figures forming the numbers $b_1, b_2, \ldots$ (the lower place figures of u). Part II contains (for the tabulated x_i) the values of Δx_i (difference between the given and tabulated x_i) that correspond for a sequence of values $x_j = c_0, c_1, c_2 \ldots$ (which is usually by far more sparse than the basic one) to a succession of values $\Delta u = d_1, d_2, \ldots$ of requisite accuracy. Part III makes it unnecessary to sum up the first approximation of u with the correction Δu [1].

Each of the tables (e) corresponds to a definite value $x_i = \overline{x_i}$.

In Part I at the intersection between the k-th column with the r-th line we place the value of x_j satisfying with the requisite accuracy the system (A) for a given $\overline{x_i}$ and for a value of u formed by suffixing to the number a_k corresponding to the k-th column, the number b_r corresponding to the r-th line.

In Part II at the intersection of the p-th column with the q-th line we place the value of Δx_i (difference between the given and the tabulated x_i) corresponding for a given $c_{q-1} < x_j \leqslant c_q$ to a certain value $\Delta u = d_p$* which marks the p-th column in the scheme. By slightly increasing the volume of the table it is often possible to choose for $\overline{x_i}$ (the tabulated value of x_i) a sequence of round values, which simplifies the determination of the values of Δx_i.

The sequence of tabulated values of $x_j = c_0, c_1, c_2, \ldots, c_l$ which mark the lines of Part II, and the succession of values of $\Delta u = d_1, d_2, \ldots$ which correspond to the columns of Part II are to be so selected as to meet the required accuracy of the values of u and not to let the highest of the $d_1, d_2, \ldots$ exceed the highest of the $b_1, b_2, \ldots$ (the values that mark the lines of Part I).

In Part III the intersection between the s-th line and the t-th column is assigned either the values $b_s - d_t$, or $b_s + d_t$, according as the value of Δu is positive or negative. By virtue of 2) the number $b_s + d_t$ can contain only one cipher more than the highest of the $b_1, b_2, \ldots$ And the figure at the high, est place is unity.

In this case the value of $b_s + d_t$ is set in the table omitting this unity, which circumstance may be indicated by an asterisk or by bold-faced type. If the corrections $\Delta u = d_t$ are negative, then one is subtracted from the numbers $a_1, a_2, \ldots$ (the columns of Part I), and the cases where $d_t < b_s$ are also marked by an asterisk.

As is obvious from the construction, the order of using the tables (e) in determining the values of u from the given x_i and x_j is as follows:

1) In table (e) marked by that value $\overline{x_i}$ which is the next lower value of x_i, in Part I we find the respective value of x_j at the intersection of an n-th column with an r-th line.

Note. If the found table (e) is marked by the value $\overline{x_i} = x_i$ (given), then the sought value of u is formed by suffixing b_r to a_n.

2) In Part II Δx_i is found in a certain line q ($c_{q-1} < x_j \leqslant c_q$) at its intersection with the column α.

3) In Part III at the intersection of line r and column α we find a number which, if suffixed (to the right) to a_n (the number marking the column in which x_j was found), will give the sought for value of u.

Note. If the number of Part III thus determined (let us denote it by $\overline{\Delta u}$) is bearing in asterisk, then the number a_n before being prefixed to $\overline{\Delta u}$ must be increased by one.

Tables of the described construction serve for deriving the values of u from system (A). Obviously, the table for determining the values of v can be plotted in a similar way. If the size of the page allows it, the tables for u and

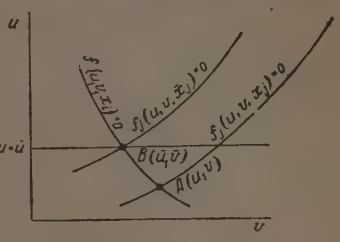
---

* The columns of Part II need not be marked by the values of Δu (just as in the case of dump scale in nomography).

v corresponding to a single tabulated value $\bar{x}_i$ are placed side-by-side (on the same page or on two pages facing each other).

Suppose that in solving the system (A) the values of u and v are known beforehand with some approximation and that the number of pair-values of the parameter p, for which the system (A) is to be solved, is comparatively high. Then the plotting of this kind of table for every pair i, j is difficult on account of the space required. Here for economy of space the following method can be recommended for the tabulation of the system (A). The operations involved are similar to those described above.

To begin with, let us put a geometric interpretation upon the method of solving the system (A) which will make the basis of our tables. Given in the plane (u, v) (see figure) two systems of curves $f_i(u, v, x_i) = 0$ and $f_j(u, v, x_j) = 0$ depending upon the parameters x_i and x_j, find the point of intersection $A(u, v)$ of two curves of these systems, corresponding to given values of x_i and x_j. We find the intersection point $B(\bar{u}, \bar{v})$ between the straight line $u = \bar{u}$ and the curve $f_i(u, v, x_i) = 0$ (α). Through the point B we draw the curve $f_j(u, v, \bar{x}_j) = 0$ (β) of the second system. From the difference $\Delta x_j = \bar{x}_j - x_j$ at point B we can find $\Delta u = u - \bar{u}$ and $\Delta v = v - \bar{v}$, or, in other words, the point A we are looking for. In case Δu is not too large, A is simply found by linear interpolation.



Tabulation on the basis of this method reduces to plotting such tables for a system of values $\bar{u}$ of a variable u, as give the solution of the equations (α) and (β), in other words, tables solving the equation $f_p(\bar{u}, v, x_p) = 0$ for all possible values of the parameter p (left-hand side), and correction tables (right-hand side).

The tabulated values $\bar{u}$ of the variable u are sufficiently far between. For the tabular $\bar{u}$ the table has the following shape.

$$u = \bar{u}$$

Left-hand side | Right-hand side

p / v	$\ldots i \ldots j \ldots$	$\ldots$	i, j / $\Delta x_j\ \Delta x_j$	$\ldots$	Δu	Δv
a_1		$\ldots$		$\ldots$	b_1	c_1
a_2		$\ldots$		$\ldots$	b_2	c_2
$\cdot$					$\cdot$	$\cdot$
$\cdot$					$\cdot$	$\cdot$
$\cdot$					$\cdot$	$\cdot$
a_m		$\ldots$		$\ldots$	b_n	c_l

(f)

In the left-hand side of the table (f) at the intersection of column q with line r we place the values x_q corresponding with the requisite accuracy to the equation $f_q(u, v, x_q) = 0$ for that value $\bar{u}$ which marks the given table and for the value $v = a_r$.

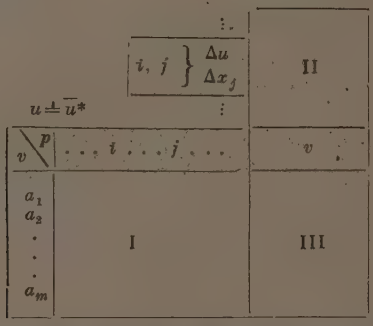
In the right-hand side of table (f) each column corresponds to some system of values i, j where $i < j$. Every such column consists of two columns (left and right). In the left one the line q contains the value Δx_j corresponding to a certain value $\Delta u = b_q$. In the right column the line q contains the value Δx_i corresponding to a certain value $\Delta v = c_q$. The sequences of values $a_1, a_2, \ldots$ and $c_1, c_2, \ldots$ are chosen to meet the required accuracy for u and v.

The order in which the tables (f) are to be used in determining the values u and v when x_i and x_j are given, is seen from the construction to be as follows. On the left-hand side of table (f) marked by the corresponding value of $\bar{u}$ we find the value of x_i in an r-th line of column i. In the corresponding line of the column j we find the corresponding value of x_j. If this value of x_j is equal (with the required accuracy) to the given one, then the number a_r marking the line r in which we have found the given values of x_i and x_j, will give us the sought for value of v, while the value $\bar{u}$ by which the given table (f) is marked gives the sought for value of u.

If the x_j thus found differs from the given value, we have to proceed as follows. On the right-hand side of table (f) the column i, j $(i < j)$ is examined for the value of Δx_j which must be contained in some line s of the left column, and in some line t of the right column. This means that $\Delta u = b_s$ and $\Delta v = c_t$.

Note. If the ratio $\Delta u / \Delta v$ differs little from unity, then we may restrict ourselves to only one column Δx_j in the right-hand side of the columns i, j.

In practice while solving the system (A) the range of variation of u, v, x_p will sometimes be found to be such that, whatever the given value $\bar{u}$, the quantity x_p will take values lying within the corresponding intervals only for certain values of the parameter p, but not for all. In such cases and if $\Delta u / \Delta v$ differs little from unity, the construction consisting of the tables of the following form is preferable.



(g)

In this construction the method employed in Parts II and III of the construction (e) for eliminating arithmetic operations is combined with that used in the left-hand part of the construction (f). Tables (g) are to be used in the following order.

In Part I, just as in the left-hand part of the tables (f), we find the given value of x_i and the corresponding value of x_j in some line r.

Note. If this value of x_j is equal with the required accuracy to the given value, then $u = \bar{u}$ and $v = a_r$.

In the Part II in the double line i, j, the line Δx_j contains the required Δx_j in some column s.

The number just above it is Δu and the number in the Part III at the intersection between column s and line r is the sought for value of v.

In the case of construction (f) and (g) if the approximate value of u and v are unknown, it will suffice to use first a table plotted for a sparse system of values u and v.

Received
19. 1. 1944.

REFERENCES

[1] L. J. Neushuler, C. R. Acad. Sci. URSS, XLIII, No. 4 (1944).

* The values $\bar{u}$ can, provided this does not involve a considerable extension of volume, are to be chosen such that the sought for values of u may be formed by suffixing to the numerals of $\bar{u}$ the numerals of Δu. This remark refers also to the construction (f).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 9

PHYSICS

ON A NEW METHOD OF ACCELERATION OF RELATIVISTIC PARTICLES

By V. I. VEKSLER

(Communicated by S. I. Wawilow, Member of the Academy, 19. VII. 1944)

As was shown in a preceding paper (1), acceleration of relativistic particles can be attained by applying a constant magnetic field. We hope to prove below that due to automatic phasing resonance acceleration is possible not only in a constant magnetic field, but also in those in which the intensity increases with time. In contrast to the Wideroe-Kerst betatron, in this case the magnetic field is designed to control the paths of the particles, while the acceleration is a result of the variable electrical field*. The advantage of the resonance accelerator over the solenoidal is that the effect of radiation (due to the motion of the particle in the magnetic field) on the acceleration does not exist in the former; another advantage is that the magnet can be constructed in the form of a thin ring, which is a very profitable feature.

Principle of Action. Consider N acceleration gaps (and a variable electric field of frequency ν and amplitude V_0 applied to them) arranged in a plane perpendicular to the magnetic field. Suppose the ring magnet produce a field which slowly increases. It is evident that in this case resonance acceleration will be attained if the energy of the particles can be made to increase synchronically with the increasing magnetic field.

Indeed, the resonance method requires the constancy of

$$T(t) = \frac{2\pi m(t)c}{H(t)e} = \frac{2\pi E(t)}{H(t)ec}$$

where $E(t) = m(t)c^2$ is the total energy of the particle. The first impression is that this requirement can be met only by choosing a particular relation between the magnetic field strength and the radius of the orbit, time, *etc.*

However, it can be proved that in order to achieve synchronism it is only necessary to observe two very general limitations:

1) variation of the magnetic field along the redius should be comparati ve ly small, *i. e.*

$$\frac{1}{H_m}\int_{r=R_{\min}}^{r=R_{\max}} \frac{\partial H}{\partial r}\,dr \ll 1 \qquad (1)$$

* In principle, of course, resonance of a higher order is possible [see (1)].

2) due to the solenoidal field the potential difference which is communicated to the particle during each revolution must be less than V_0. In the case of $N=2$ this condition may be expressed as follows:

$$\left(\frac{\partial H}{\partial t}\right)_{\max} \ll \frac{4\pi V_0}{T_\lambda^2 c} \qquad (2)$$

In any accelerator in which these simple requirements are fulfilled*, synchronization will be automatic and independent of the relation between the growth of the magnetic field and the time.

The mechanism by which the constancy of the period of revolution of the particles in their orbits is maintained can be explained by the automatic phasing action; this will be clear on considering the formula for the time T_n of the n-th revolution of the particles in the magnetic field:

$$T_n = \frac{2\pi\left\{e\left[\sum_{i=1}^{i=n}(V_0\cos\varphi_i + u_i)\right] + m_0c^2(k+1)\right\}}{N\overline{H}_n ec} \qquad (3)$$

Here $k=\frac{eV_{in}}{m_0c^2}$; V_{in} is the potential difference corresponding to the initial velocity of the particles; $\overline{H}_n$ is the average value of the magnetic field during the period of the n-th semi-revolution; $V_0\cos\varphi_i$ is the potential difference which accelerates the particle on its i-th traversal of the acceleration gap; u_i is the potential difference the particle receives during the i-th semi-revolution as a result of the effect of the solenoidal gradient.

It will be seen from formula (3) that the time for each subsequent revolution of the particle in the magnetic field depends on the potential difference which accelerated the particle during the preceding cycles. Therefore, if the initial condition

$$T_0 = \frac{T_\lambda}{2} = \frac{2\pi m_0c^2(k+1)}{NH_0ec}, \text{ where } N \geqslant 2$$

is fulfilled, then each time the increase of the period of the n-th revolution (due to the n-th traversal of the gaps by the particle) is larger than the decrease of the period (due to the increase of the magnetic field during the n-th revolution), the particle will approach the following gap later than within $T_\lambda/2$, and therefore on the $n+1$-th traversal the field will be weaker than during the n-th revolution. On the contrary, if the energy increment (during the n-th acceleration) is less than the increase of the magnetic field during the following time interval, then T_n will be less than $T_\lambda/2$, the particle arrives earlier and therefore traverses a more intensive field than that which was effective during the previous acceleration.

As the magnetic field is continually growing, any deviation of the period of revolution from the resonance period, which may arise during the i-th revolution, decreases.

It can easily be shown that the increment of the solenoidal field during each revolution decreases as the number of cycles increases; thus, in accord with the foregoing the effect of the solenoidal acceleration will in general decrease as the number of revolutions is increased.

It will thus be seen that solenoidal acceleration is not an obstacle for resonance acceleration of relativistic particles in an increasing magnetic field.

---

* In order that the beam be strongly compressed in the direction perpendicular to the orbit plane, it is sufficient that the magnetic field decreases to the borders for some per cent only. Therefore condition (1) does not fully correspond to the focussing requirements.

These considerations are easily confirmed by a mathematical analysis of the phasing process. If we take, for instance, the case $N=2$, we obtain for $\frac{dT_n}{dn}$ the expression

$$\frac{dT_n}{dn}=\frac{\pi V_0\cos\varphi_n}{\overline{H}_n c}+\frac{\pi u_n}{\overline{H}_n c}-T_n\frac{1}{H_n}\frac{dH_n}{dn} \tag{4}$$

Taking into consideration that

$$\varphi=\frac{2\pi}{T_\lambda}\left(\sum_{i=1}^{i=n}T_i-\frac{nT_\lambda}{2}\right) \quad \text{and} \quad t=\sum_{i=1}^{i=n}T_i$$

we can express u_n in terms of R_n and H_n:

$$u(t)=\frac{1}{2c}\frac{\partial\Phi}{\partial t}=\frac{1}{2c}\pi(R_n^2-R_{\min}^2)\frac{\partial H_n{}^*}{\partial t}$$

The expression for R_n in terms of T_n is

$$R_n^2=\frac{T_n^2c^2}{\pi^2}-\left[\frac{m_0c^2}{\overline{H}_n ec}\right]^2c^2 \tag{5}$$

Finally, assuming that

$$\varphi_n=\psi_n+\alpha_n \tag{6}$$

where ψ_n weakly depends on n and α_n is a small quantity (such that $\sin\alpha_n\approx\alpha_n$), we obtain the following equation for α_n:

$$i\frac{d^2\alpha}{dn^2}\frac{d\alpha}{dn}+m\frac{d^2\alpha}{dn^2}-A'\frac{d\alpha}{dn}\alpha-A\alpha=B\frac{d\alpha}{dn}+k\left(\frac{d\alpha}{dn}\right)^2+g\ ^{**} \tag{7}$$

where i, m, A, A', B, k and g are coefficients which depend on certain constants and quantities which slowly vary with n.

Assuming that

$$i\frac{d^2\alpha}{dn^2}\frac{d\alpha}{dn}\ll\frac{d^2\alpha}{dn^2}m; \quad A'\frac{d\alpha}{dn}\alpha\ll A\alpha; \quad k\left(\frac{d\alpha}{dn}\right)^2\ll B\frac{d\alpha}{dn}$$

and considering in a first approximation that the coefficients A, B, m, g are constant, we obtain the solution of the simple field equation in the form

$$\alpha_n=A_0e^{-\delta n}\sin(\gamma n+\Omega_0)+\mu$$

where δ and μ weakly depend on n. Insertion of this (or the more exact solution ** in equation (7) justifies the neglect of the non-linear terms in (7). Thus, as n increases, the phase approaches a certain limiting value, *i. e.* automatic phasing does really exist. It can be shown that for the exact as well as simplified solution condition (2) follows from the requirement that the limiting value of α_n be much less than π. Although we considered that H is constant along the radius, it is evident that a slow variation of H will not alter our result.

---

* Here $R_{\min}$ is the inner radius of the accelerator magnet which is a ring with an area $S=\pi(R_{\max}^2-R_{\min}^2)=\frac{\pi R_{\min}^2}{\beta_{in}^2}(1-\beta_{in}^2)$ where β_{in} is the initial value $v/c=v_{in}/c$.

** An equation of the same type exists, of course, for any other value of N.

*** Assuming that the coefficients A, B, m depend on H_n (or otherwise on n) the following solution can easily be obtained

$$\alpha_n=c_1(2\sqrt{H_n})^{-(A-1)}I_{A-1}(\sqrt{BH_n})+c_2(2\sqrt{H_n})^{-(A-1)}I_{-(A-1)}(\sqrt{BH_n})+\mu.$$

In conclusion, it should be mentioned that automatic phasing will also compensate the «untuning» resulting from radiation due to the motion of the relativistic particle in the magnetic field *.

It is quite possible that as a result of this circumstance highly energetic particles may be obtained by this method.

From the foregoing it simply follows that all maximum energy which could be attained depends on the magnetic field in the following manner:

$$E_{\max} = m_0 c^2 (k+1) \frac{H_{\max}}{H_0} = \frac{H_{\max} e}{\sqrt{1 - \left(\frac{1}{k+1}\right)^2}} R_{\min}.$$

P. N. Lebedev Physical Institute.
Academy of Sciences of the USSR.

Received
8. VII. 1944.

REFERENCES

[1] V. Veksler, C. R. Acad. Sci. URSS, XLIII, No. 8 (1944).

* This compensation will continue to exist until the «resonance» potential difference which gradually decreases finally reaches V_0. Obviously, this practically means that the upper limit will reach the region of huge energies.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 9

ELECTRICAL ENGINEERING

DETERMINATION OF RESONANCE CURVE MAXIMA FOR A SYMMETRIC HOMOGENEOUS RECURRENT CIRCUIT

By P. L. KALANTAROV and L. A. ZEITLIN

(Communicated by V. F. Mitkevitch, Member of the Academy, 20. I. 1944)

1. Homogeneous recurrent circuits are very often used as filters in communication engineering. The most important characteristic of such a filter is its resonance curve, which brings out the dependence of the current strength in the terminal impedance upon the frequency when the emf or the generator feeding the filter is constant. The losses in the filter circuits are usually very small and these circuits may be taken to be composed of purely reactive elements. The terminal impedance through which the circuit is closed, and the internal impedance of the generator feeding the circuit will here be considered as pure resistance.

A. G. Lurje (1) has shown that if the above conditions are fulfilled and, besides, the internal and terminal impedances of the generator are equal, the intensity of the current in the terminal impedance attains its maximum when the iterative impedance of the current is equal to the terminal impedance. In fact, the whole external circuit is in this case equivalent to the terminal impedance and the generator is closed through an external impedance equal to its internal impedance. It is known that when the external and internal impedances are equal, the generator gives out into the external circuit the highest proportion of its power output—a half of the total power it develops. As the losses in the filter circuit are disregarded, the power absorbed in the terminal impedance under the above-mentioned conditions is likewise a maximum and so is also the current strength in the terminal impedance which is then equal to the current strength in the generator. The latter, it may here be remarked, coincides in phase, under the considered regime, with the emf of the generator, and, accordingly, there is resonance in the circuit.

From what has been said here it follows that the problem of drawing the maximum resonance curve of a symmetrical homogeneous circuit reduces to the determination of frequencies at which the iterative impedance of the circuit is equal to its terminal impedance. All maxima of the current have here the same value, which is equal to the emf of the generator divided by the twofold terminal impedance.

2. In an earlier paper (2) the authors have shown that for the iterative impedance of the described circuit to be equal to the terminal impedance r, which in our case is a purely active one, one of the following conditions must be satisfied:

1) $\frac{B_n}{C_n}=\frac{B_1}{C_1}=Z^2$ (first kind)

2) $B_1=0, \quad C_1=0$

3) $F_n(a)=F_n(2A_1)=0$ } (second kind)

where A_1, B_1, C_1 are complex constants of a link, considered to be symmetrical four-pole networks, and B_n, C_n are the respective constants for the whole circuit composed of n similar symmetrical links. The shape of the function $F_n(a)$ and its roots are given in the above-mentioned paper (2).

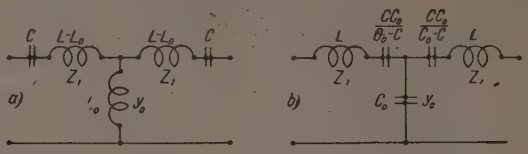


Fig. 1

3. Let us consider a very usual type of circuit, namely, one consisting of symmetrical T-shaped links, as represented in Fig. 1. For this kind of link we have, as is well known,

$$A_1 = 1 + Z_1 Y_0; \quad B_1 = 2Z_1 + Z_1^2 Y_0; \quad C_1 = Y_0$$

In the case of inductive coupling (Fig. 1, *a*) we introduce the notation

$$\frac{1}{\sqrt{LC}} = \omega_0; \quad \frac{\omega}{\omega_0} = \eta; \quad \sqrt{\frac{L}{C}} = \rho; \quad \frac{L_0}{L} = k$$

where ω_0 is the proper frequency of each of the coupled elementary circuits making up the recurrent circuit under consideration; ρ is the wave impedance of the elementary circuit; k, the coefficient of coupling; η, the ratio of the frequency of emf of the generator to the resonant frequency of the elementary circuits. Then we have

$$Z_1 = j\left(\omega L - \omega L_0 - \frac{1}{\omega C}\right) = j\rho\eta\left[(1-k) - \frac{1}{\eta^2}\right]$$

$$Y_0 = \frac{1}{j\omega L_0} = -\frac{j}{\rho\eta k}$$

and putting $1/\eta^2 = x$, shall after some simple transformations obtain

$$A_1 = \frac{1-x}{k}; \quad \frac{B_1}{C_1} = \frac{\rho^2}{x}[k^2 - (1-x)^2]$$

In the case of capacitive coupling (Fig. 1, *b*) we arrive at the same expressions by putting $C/C_0 = k$ and $\eta^2 = x$ and retaining the same notation for ω_0, η and ρ for A_1 and B_1/C_1.

4. Using the obtained expression for B_1/C_1 and introducing the damping $d = r/\rho$ of the input and output circuits, we find from the condition $B_1/C_1 = r^2$ (first kind) that

$$x^2 - (2-d^2)x + (1-k^2) = 0$$

For $d < 2$ and $k > d\sqrt{1 - \frac{d^2}{4}}$ this equation yields two positive values of x, and so we have found two positive values for the frequency, to which there correspond two first kind maxima in the resonance curve. These maxima fuse together when the coupling coefficient takes critical value $k = k_c = d\sqrt{1 - \frac{d^2}{4}}$, but if $k < k_c$, there is only one maximum of special kind, defined by the equation $3x^2 - (2-d^2)x - (1-k^2) = 0$, and this maximum is lower in value than those of the first and second kind. We will not here dwell upon it because in the filters $k > k_c$ is so chosen as to bring all maxima to a single value which should, moreover, be the highest possible one.

5. Proceeding to the conditions of the second kind, let us note that the conditions $B_1=0$ and $C_1=0$ drop out, because in the two particular cases considered $C_1=Y_0\neq 0$. From the condition $F_n(a)=0$ we find $(n-1)$ values

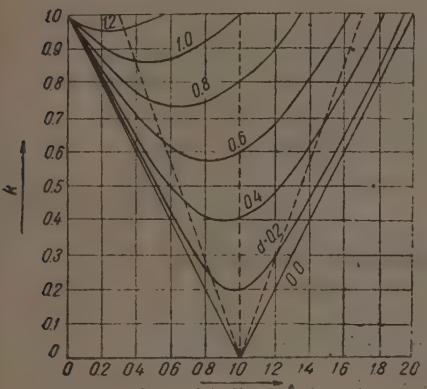


Fig. 2

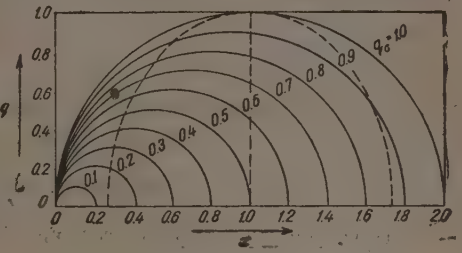


Fig. 3

of A_1. As in our case $x=1-A_1k$, the conditions of the second kind yield $(n-1)$ second kind maxima. Thus, for $k>k_c$ the total number of maxima (the first and second kind taken together) will be $(n+1)$, which is one more than the number of links of the recurrent circuit and equal to the number of its elementary circuits.

6. The values of x corresponding to the maxima of the resonance curve can be determined graphically. For this purpose we plot in coordinates (x, k) a family of hyperbolas defined by the equation $x^2-(2-d^2)x+(1-k^2)=0$, and a bundle of straight lines defined by the equation $x+A_1k=1$. In Fig. 2 these lines (dotted) are drawn for $n=4$.

Drawing a straight line parallel to the axis of abscissae and separated from it by a distance equal to the given value of k, we shall find all values of x, hence all frequencies, that correspond to the maxima of the resonance curve. They are the abscissae of the intersection points of this straight line with the family of hyperbolas and with the bundle of straight lines passing through the point (1, 0).

Instead of k we can introduce a coefficient q linked with k by the relation $q^2=1-k^2$. Then the conditions of the first and second kind will for the circuits considered be reduced, respectively, to the relations

$$(x-q_c)^2+q^2=q_c^2, \quad \left(\frac{1-x}{A_1}\right)^2+q^2=1$$

where $q_c=\sqrt{1-k_c^2}=\frac{2-d^2}{2}$ is the critical value of q.

Expressed in coordinates (x, q) the first equation is a family of circles, and the second, a family of ellipses (Fig. 3). By plotting these curves all frequencies corresponding to the maxima of the resonance curve can be determined as described above.

Received
20. I. 1944.

REFERENCES

[1] A. Lurje, Arch. f. Elektr., XXIII, 5, 485 (1930). [2] P. L. Kalantarov and L. A. Zeitlin, C. R. Acad. Sci. URSS, XLIII, No. 7 (1944).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 9

CHEMISTRY

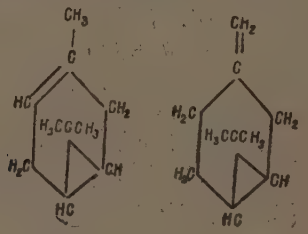
RAMAN EFFECT INVESTIGATION OF TURPENTINE FROM *PINUS SILVESTRIS*

By G. V. PIGULEVSKY and A. T. RYSKALCHUK

(Communicated by A. E. Arbusov, Member of the Academy, 5. III. 1944)

According to recent data [1, 2] the common turpentine is made up of *d*-α-pinene, *d*-Δ³-carene, *l*-terpene and some high-boiling components. But our knowledge of its composition is still unsatisfactory, even though more than one thorough investigation has been carried out in this field.

The nature of *l*-terpene is unclear. Δ³-carene is difficult to isolate and there is reason to suppose that it is attended by admixtures. Moreover, tests of carene isolated from *Pinus longifolia* have shown it to be no homogeneous compound. According to Dupont et Joffre [4], Nevgi and Jatkar [5], Δ³-carene contains a hydrocarbon as admixture, with a frequency of 1640 cm^{-1}, which corresponds to the double bond in the side-chain. The products arising from ozonization of the carene fraction have been found by Joffre [3] to contain fumaric aldehyde and fumaric acid in considerable quantities. From this data and from spectrographic studies [4] it is argued that carene is a mixture of Δ³-carene and β-carene.



Δ³-carene β-carene

For carene fractions of the turpentine obtained from *Pinus silvestris* no such data are available.

The present authors have made an attempt to investigate the turpentine from *Pinus silvestris* spectrographically—by means of the Raman effect. For this purpose this pine's turpentine was distilled up into fractions and each fraction was studied spectrographically.

Six fractions were thus separated and found to possess the following properties (Table 1).

The data of this table on the rotation of the polarization plane, specific weight and refraction index show a variation typical of many turpentines among the number investigated by Russian scientists.

Table 1

Nos. of fractions	B.p., °C	α_D	$\frac{\alpha_F}{\alpha_C}$	D_4^{15}	n_D^{15}
1	157—158	+12.86°	1.96	0.8629	1.46849
2	157—158	+12.60	1.97	0.8628	1.46790
3	158—159	+12.00	2.00	0.8625	1.46761
4	159—160.5	+10.75	2.23	0.8627	1.46839
5	160.5—162	+10.76	2.03	0.8636	1.46908
6	164.5—165.7	+ 3.65	2.66	0.8631	1.46958
7	166—169	+ 0.22	—20.2	0.8626	1.47185
8	169—172	— 4.65	1.44	0.8624	1.47264
9	172—174	— 9.45	1.51	0.8627	1.47362
10	174—176	—14.15	1.81	0.8640	1.47489

Table 2

Raman Spectrum of Turpentine from *Pinus silvestris* ($\Delta\nu$)

Fraction No. 1	Fraction No. 2	Fraction No. 4	Fraction No. 5	Fraction No. 7	Fraction No. 8
157—158° C	157—158° C	159—160.5° C	160.5—162° C	166—169° C	169—172° C
138	143(1)	139(2)			
209	204(1)	204(1)			
261	263(2)	259(2)	262(1)		
308	308(1)	305(1)	310(1)		
					344(1)
390	390(1)	387(2)			
426	427(2)	424(2)	426(2)	419(2)	424(2)
			432(2)		
		462(1)			
471		480(1)			502(1)
562	569(2)	565(2)		567(2)	569(2)
619	620(1)	619(1)	620(1)		
667	669(3)	665(3)	668(3)	672(3)	678(3)
		713(1)	715(1)	712(3)	713(3)
				759(1)	759(2)
776	774(2)	770(2)	771(1)		
823			823(1)	819(2)	820(1)
842	845(2)	843(2)	844(1)	844(1)	840(1)
905		893(1)			
	942(1)	940(1)	944(1)	939(1)	934(1)
1044	1044(1)	1044(2)	1044(1)		
1085	1083(1)	1086(2)	1087(1)		
1128	1124(1)	1125(1)	1126(1)		
				1154(1)	1156(1)
1175	1170(1)	1173(1)			
1219	1213(1)	1213(1)	1218(1)	1218(1)	1211(1)
1268	1266(1)	1262(2)	1266(1)		
1305		1306(1)	1310(1)	1302(1)	1308(1)
1328	1333(1)	1328(2)	1334(1)		
1377	1382(1)	1376(2)	1377	1370(2)	1375(2)
1430 } 1462 }	1441(5)	1444(3)	1441(4)	1434(4)	1439(4)
1660	1660(3)	1658(2)	1662(2)	1638(1)	1643(2)
				1677(1)	1681(2)
	2925(2)	2939(1)			

Fractions 1, 2, 4, 5, 7 and 8 were subjected to spectrographic investigations. The Raman spectrum was taken by means of a Zeiss three-prism spectrograph. Mercury line 4358.3 was used as a source of excitation of Raman lines.

Slit width, 0.1 mm. Exposure for 20 to 30 hours. A solution of sodium nitrate served as a filter. The results are listed in Table 2.

Fractions 1, 2 and 4 give a pinene spectrum ([6]). Fraction 5 is somewhat different from these. A series of frequencies characteristic of pinene disappear and some typical of Δ^3-carene appear instead: 432, 715 and 832 cm^{-1}. Yet the double bond is represented by a pinene frequency, *viz.* 1662 cm^{-1}. Fractions 7 and 8 give a spectrum typical of Δ^3-carene ([6,7]). The only difference is that the 1105—1111 cm^{-1} frequency* is absent in fraction 8. The double bond is represented by two frequencies of equal intensity, 1643 and 1681 cm^{-1}. Thus, two fractions with $\alpha_D + 0.22°$ and α_D—4.65° give the same Raman spectrum characteristic of carene.

Comparisons of our data with those available in literature suggest the presence of β-carene and Δ^3-carene among carene fractions of the turpentine from *P. silvestris*.

From the absence of frequencies foreign to Δ^3-carene and the fact that the rotation of the polarization plane is toward the left, one may infer the existence of a laevorotating form of Δ^3-carene.

Evidence supporting this inference has been obtained by continuing the distillation of carene fractions. In this way the authors obtained a fraction with $\alpha_D = -8.85°$ (b. p. 57—58° at 11 mm). The Raman spectrum of this fraction contained no frequencies foreign to carene and was very characteristic of this substance.

Raman Spectrum of the Fraction with $\alpha_D = -8.85°$

347(0.5); 419(1); 502(1); 567(1); 676(2); 711(2);
758(2); 787(0.5); 818(1); 934(0.5); 1153(1); 1213(1);
1308(1); 1375(3); 1426—1445(3); 1637(3); 1681(3)

β-carene is probably present among the carene fractions in the dextrarotating form.

Along with β-carene and Δ^3-carene *l*-terpene** enters into the composition of turpentine. The structure of this compound must be established by spectrographic investigations.

The authors express their heartiest thanks to Prof. E. Gros for advice and assistance.

Received
5. III. 1944.

REFERENCES

[1] Б. Арбузов, ЖРФХО, **61**, 247 (1927); ЖПХ, **7**, 758 (1934). [2] В. Крестинский, С. Малевская, Ф. Солодкий, ЖПХ, **5**, 950 (1932). [3] M-lle Joffre, Bull. Inst. du Pin, No. 16,17 (1931). [4] G. Dupont et M-lle Joffre, Bull. Inst. du Pin (1932). [5] G. V. Nevgi and S. K. Jatkar, Journ. Indian Inst. of Sci., Ser. A, **17**, 189 (1934). [6] W. Rogie Angus, Proceed. of Indian Academy of Sci., Sec. A, **8**, 529 (1938). [7] G. Dupont, P. Daure, J. Allard, Bull. Soc. Chim., **49**, 1481 (1931).

* Two figures mark the range within which the true frequency is confined.

** By the kindness of Prof. B. A. Arbusov the results of his unpublished work on turpentine residues after the separation of pinene have become known to the authors. This commercial product of the Kuskov works has been found to contain *d*-Δ^3-carene, *l*-terpene and terpinolene. Careful investigation has shown that its *l*-terpene is a mixture of *d*-Δ^3-carene and a strongly racemized *l*-limonene. It is hard to decide at this juncture in what measure these results may bear upon the study of the common turpentine, as there is no identity of nature between the two products.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 9

CHEMISTRY

ON THE SULPHUR COMPOUNDS IN PETROLEUMS

By I. J. POSTOVSKY, N. P. BEDNYAGHINA and M. A. MIKHAILOVA

(Communicated by S. S. Nametkin, Member of the Academy, 6. III. 1944)

In his book «The Chemistry of Petroleum» S. S. Nametkin (3) has the following to say about the sulphur compounds in petroleum distillates: «There is no doubt that a large number of the simplest sulphur compounds detected in petroleum distillates are not present in the original oil itself at all, but on the contrary, only form during its distillation». In this way the formation of the sulphur compounds may be connected with the disintegration of complex sulphur-containing products or with their synthesis by the interaction of elementary sulphur with hydrocarbons of the petroleum. Similar views have been advanced by other scientists as well(4,5).

Especially interesting is the problem of the origin of the simplest thiophene compounds in the distillation of sulphur-containing petroleums. If the presence in the original petroleum or in the petroleum bitumens of complex sulphur compounds (6, 7), perhaps even of cyclic structure (6), yielding thiophenes upon thermal disintegration (7), is confirmed, this will furnish new data for the geochemistry of petroleum, that will shed light on the processes of the transformation of the parent substances of the oil into the sulphur-containing petroleums of the present time.

For an investigation of this problem we chose the sulphur-containing petroleums of the Chusovo, Ishimbay and Krasnokamsk oil-fields in the Urals. It is a simple matter to detect the presence of thiophene compounds in the clear distillates of these petroleums by the qualitative indophenine test. Chusovo petroleum (from oil well No. 101) already gave a definite reaction for thiophenes in the low-boiling (120—130 °C) fractions (temperature within the liquid 170—180 °C). The Ishimbay and Krasnokamsk petroleums (average samples taken from the collector) gave this reaction in higher-boiling (220—225 °C) fractions (temperature within the liquid 280 °C).

In order to elucidate whether the presence of the thiophene compounds in the distillates is only a result of the thermal treatment of the petroleum, we proceeded as follows. From the sulphur-containing petroleums the asphaltenes were removed by precipitation with petroleum ether and then the etherial filtrate was subjected to chromatographic adsorption in a glass tube filled with silica gel.

Upon illuminating the column with filtered ultra-violet light from a mercury quartz lamp three sharply defined characteristic zones are revealed [«ultrachromatograms» (8)]: a small practically non-fluorescing, blackish-brown zone on the top, another smallish zone with a bright bluish-violet fluorescence on the bottom, and a zone of somewhat larger dimensions, fluorescing distinctly with a greenish yellow light, in between.

The blackish-brown upper layer of the chromatogram, containing bituminous substances inextractable from silica gel in the cold either by petroleum ether or benzene, was scraped out of the column and then extracted in a Soxhlet apparatus with benzene. After the benzene was distilled off in a vacuum, a thick,

black practically odourless oil (from 8 to 12 per cent of the petroleum) remained. The two yellow and violet fluorescing layers that remained in the column were washed from the top with petroleum ether until the violet fluorescing lower layer vanished. The wash petroleum ether was distilled in vacuum at 60° C, after which a colourless mobile oil («light oil») smelling of kerosene (from 70 to 75 per cent of the petroleum) remained. The middle (yellow) layer that remained in the column was washed with benzene in the cold until it ceased fluorescing. After distillation of the wash benzene, a yellowish highly viscous oil («heavy oil») smelling slightly of turpentine (from 6 to 8 per cent of the petroleum) was obtained*.

Each fraction obtained in the cold was tested for thiophenes by the indophenine reaction before and after a distillation (up to 300 °C). Not one of the cold fractions gave a positive reaction; only upon distillation of light and heavy oils was it possible to obtain a distinctly positive reaction for thiophenes in the distillates (see table).

The Temperature Range within which the Fractions upon Distillation Give the Indophenine Reaction in the Distillates
(the temperature within the liquid)

Fraction / Petroleum	Light oil		Heavy oil		Bituminous substances	Asphaltenes
	temperature interval (in °C)	*	temperature interval (in °C)	*		
Chusovo	125—130	+	125—300	++	no reaction	no reaction
Ishimbay	200—250	±	150—250	+		
Krasnokamsk	no reaction		200—250	±		

* The intensity of the colour reaction is indicated by the following conventional signs: ++very distinct; + distinct; ±weak. The tests were carried out with a solution in pure benzene.

On the basis of the experimental data given in the table, one may draw the conclusion that the simplest thiophene compounds are not present in the original samples of petroleums investigated, but are formed during distillation.

The low temperature (125—150 °C) at which the thiophenes make their appearance in the distillates speaks against their formation by synthesis from elementary sulphur and hydrocarbons during the distillation of the petroleum. If such synthesis were to take place, one should expect the largest evolution of hydrogen sulphide in those cases where the indophenine reaction was found the most intensive, whereas the Chusovo petroleum, whose distillates gave the most definite reaction for thiophenes, gives off ten times less hydrogen sulphide than Ishimbay petroleum (9), for example, which gave this reaction only weakly. Apart from this, as is seen from the table, the part of the petroleum with the highest sulphur content—the bituminous substances and asphaltenes—did not give the indophenine reaction at all during thermal disintegration**, although the quantity of hydrogen sulphide that is given off during their disintegration comprises more than half of all the hydrogen sulphide that forms during the distillation of the petroleum (9). Thus, it may with great probability be concluded that the thiophene compounds (which we will term «thiophenogens») are formed during the distillation of the petroleum as a result not of synthesis, but of di-

* Application of the ultrachromatograph in the investigation of petroleum compounds deserves attention, in our opinion, notwithstanding the laboriousness of the experimental procedure; thus, for, example, upon repeating the procedure one is able to obtain by a second ultrachromatogram fractions with a high content of sulphur compounds (see below).

** Thermal disintegration of the heavy oil, bituminous substances and asphaltenes was carried out by heating to 300°C after their solution in oil known not to contain thiophene-forming substances.

sintegration of the complex sulphur-containing compounds.

Characteristic of thiophenogens is the important fact that they do not possess the ability to be firmly fixed on an adsorbent. By repeated washing in the cold with petroleum ether one may gradually remove all the thiophenogens from the heavy oil adsorbed by silica gel, some of the oil with a high total sulphur content remaining on the adsorbent just the same. Thus, for example, in the sample of Chusovo petroleum (oil well No. 105, sulphur content 4.5 per cent), the total sulphur content in the heavy oil from the chromatogram was 7.5 per cent, and after chromatographing four times the oil remaining, although it contained 8.5 per cent of total sulphur, did not give a positive indophenine reaction in the distillates. In the fraction removed by working with petroleum ether, the total content of sulphur was 6.6 per cent after distillation of the solution. Hence it follows that along with the thiophenogens, relatively weakly adsorbed by the silica gel, there remain in the heavy oil complex sulphur compounds of a different (unsaturated?) nature, firmly adsorbed by the silica gel.

In order to shed light on the chemical nature of the primary sulphur compounds of petroleum (thiophenogens and others), analytical research will still have to be continued for some time. That is why, simultaneously with the conduction of such an extremely laborious research, it is expedient to approach the analysis of the chemical nature of the primary sulphur compounds of petroleum from the point of view of their possible genesis as well. As regards thiophenogens, it is interesting to mention the following observations. In Chusovo petroleum, in which it is so easy to prove the presence of thiophenogens, porphyrines are entirely absent([1], [2]), whereas the liquid bitumens extracted from the oil-bearing stratum of oil well No. 303 (3rd series of the Kungur terrace, Solikamsk*), which showed an exceptionally high content of the vanadium complex of the porphyrines, notwithstanding the considerable content of total sulphur (4.4 per cent) upon the distillation of cold fractions did not give even traces of thiophenes, *i. e.*, did not contain thiophenogens. Such a contrast between thiophenes and porphyrines—the given examples are extreme cases—has already been observed on more than one occasion in petroleums and bitumens. This contrast becomes comprehensible if one assumes the transformation of the pyrrol cycles of the porphyrines into thiophene cycles according to the principle of mutual transformations of heterocycles first discovered by J. Yuryev([10])**. That is why the possibility is not excluded that the chemical structure of thiophenogens generally resembles the structure of porphyrines. On this connexion the observations made by Treibs ([12]), that the bitumen of Vallgau-shale (Bavaria) contains a unique and to all appearances a sulphur-containing porphyrine, deserves attention.

The Urals Branch of the Academy of Sciences of the USSR.
Laboratory of Organic Chemistry.
Kirov Ural Industrial Institute.

Received
6. III. 1944.

REFERENCES

[1] И. Я. Постовский и сотр., Хим. тв. топл., VI, 459 (1935). [2] И. Я. Постовский, Сов. геология, 1941, стр. 133. [3] С. С. Наметкин, Химия нефти, 1934, стр. 208; 1939, стр. 259. [4] J. v. Braun, ZS. ang. Chem., **44**, 661 (1931). [5] G. Hradil, Petroleum, **25**, 431 (1929). [6] E. Wernicke, Brennst. Chem., **20**, 124 (1939). [7] R. Potonié, Pharm. Ztg., 78, 908 (1933). [8] P. Karrer, K. Schöpp, Helv. chim. Acta, **17**, 693 (1934). [9] В. Г. Плюснин и сотр., Нефт. хоз., **20**, № 2 (1939). [10] Ю. К. Юрьев, ЖОХ, VI, 1670 (1936). [11] И. М. Губкин, Учение о нефти, 1938, 2 изд., стр. 452. [12] A. Treibs, Lieb. Ann , **517**, 180 (1935).

* We wish to express our thanks to N. P. Gerassimov (of the city of Molotov) for furnishing us with the samples of stratum.

** For such a transformation the necessary conditions are present in nature: hydrogen sulphide from hydrogen sulphide fermentation, the contacting influence of strata, protracted action in the course of geological periods in the presence of pressure reaching several hundred atmospheres ([11]).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 9

GEOCHEMISTRY

FORMULAE FOR DETERMINING NEPHELINE AND PYROXENES IN APATITE-NEPHELINE ORES OF THE CHIBINY TUNDRAS

By B. N. MELENTIEV

(Communicated by A. E. Fersman, Member of the Academy, 15. II. 1944)

Between the chief components making up the rocks of the ijolite-urtite complex there exists a definite relationship characterized by a curvilinear surface in the constitution diagram (1). Some portions of this surface may be solved with a definite approximation univocally in the system of Cartesian coordinates.

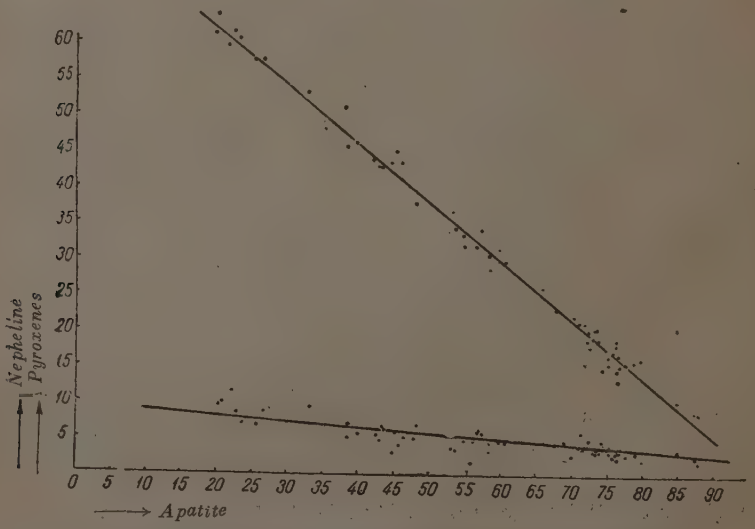


Fig. 1. Diagram showing the mutual relation in apatite-nepheline rocks

In order to facilitate the solution of this problem one can have recourse to a simple diagram, in which the apatite content is plotted along the abscissae, and the nepheline and pyroxene content, along the ordinates. The points apa-

tite-nepheline and apatite-pyroxene give two straight lines inclined at a certain angle to the abscissae. The equation of these straight lines being known, it is possible to determine their parameters, *i. e.* the ratio of phases constituting the coordinates of our system (Fig. 1).

The easiest to determine in these rocks is apatite, since no other phosphorus-bearing minerals have been so far established. Moreover, the determination of P_2O_5 itself is a simple and rather accurate matter ($2\sigma = \pm 0.28$ per cent according

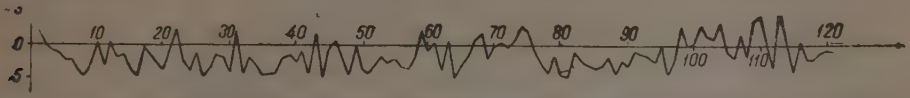


Fig. 2. Curve showing the deviations of the calculated amounts of nepheline in the samples of apatite-nepheline rocks from the result of analysis

to 197 observations). The determination of other mineral components by the usual chemical methods is not possible at present, and may be solved only by the methods of phase analysis. At the same time, the establishment of a linear relationship between the components of the ore makes it possible to determine the content of the rest of components from the content of the first which is known.

Accordingly, our equations have been built for P_2O_5.

The equation for determining nepheline has the following form:

$$x = (95.8 - 2.451\ a) \cdot 0.834 - 0.5 \qquad (1)$$

where x is the quantity sought for (nepheline content in per cent); a is the P_2O_5 content in the rock; 2.451 is the coefficient of conversion of P_2O_5 to apatite (deduced from ten complete analyses of the Chibiny apatites); 0.834 is the tangent of the angle of incline of the straight line to the abscissae, and 95.8 and 0.5 are constants.

The equation for determining pyroxenes is as follows:

$$y = (142.9 - 2.451\ a) \cdot 0.056 \qquad (2)$$

Upon the usual simplifications, the terms (1) and (2) are reduced to the form:

$$x = 79.4 - 2.004\ a = 79.4 - 0.834\ b \qquad (3)$$

$$y = 8.0 - 0.137\ a = 8.0 - 0.056\ b \qquad (4)$$

where b is the apatite content of the ore.

As the upper limit of applicability of these formulae to the ore may be taken a P_2O_5 content equal to 39.6 per cent, and as the lower limit, a content of 8.0 per cent. In the latter case, if we examine the diagram «composition — property» (1) of the ijolite-urtite system, it will be seen that the nature of the rocks changes sharply.

So the simple linear equation given above cannot any longer be applied to solve the states of equilibrium of the components existing here. To check up the correctness of the method chosen for determining nepheline in the ore, we used 138 samples, in which a determination of Al_2O_3 and P_2O_5 was carried out.

According to Al_2O_3 the nepheline content was calculated starting from an average alumina content equal to 32.29 per cent. Thirty samples were excluded from the computations, since the divergence between the nepheline content calculated and that obtained from the Al_2O_3 content exceeded the range of variation adopted, *viz.* ± 5.0 per cent for nepheline, or 1.5 per cent for Al_2O_3.

The mean arithmetic deviation (Fig. 2) was 1.6 per cent nepheline. This may be accounted for by the fact that, according to equation (3), the content of nepheline is established, while the chemically determined amount of Al_2O_3 gives the total alumina content in all the minerals of the rock.

A calculation of the mean quadratic deviation according to these 108 determinations gave a variation of ± 2.2 per cent of nepheline, which makes up 0.71

per cent of Al_2O_3. Considering that in the determination of Al_2O_3 in the nepheline concentrate the error of analysis is ± 0.50 per cent [2], a deviation of the order of 0.7 per cent is not a large one.

Table 1 shows the distribution of errors from which it will be seen that 90 per cent of all the deviations fit within $\pm 2\sigma$.

Table 1

Distribution of Errors

Range	Limit value of deviation	Number of deviations	Per cent to total number of observations
$\lvert m\rvert > 3\sigma$	6.6	0	0
$2\sigma < \lvert m\rvert < 3\sigma$	6.6	11	10.1
$\sigma < \lvert m\rvert < 2\sigma$	4.4	38	35.2
$\lvert m\rvert < \sigma$	2.2	59	54.7

Table 2 shows the number of samples fitting within the range of deviations adopted (± 5.0 per cent), and the distribution of these samples according to the textural characters of the rocks.

Table

Distribution of the Samples Studied According to Textures

Texture of rocks	Number of samples of the given texture	Number of samples in which the deviation according to nepheline was not over $\pm 5.0\%$	Per cent to the number of samples of the given texture	Per cent to the total number of samples (138)
Spotted	35	31	88.6	22.4
Lenticular-banded . . .	77	63	81.8	45.7
Reticulate	23	11	47.8	8.0
Breccia-like	3	3	100.0	2.2
Total . . .	138	108	—	78.3

Thus, it may be considered that the above equations will make it possible to solve a number of practical and theoretical problems in the study of the apatite-nepheline rocks of the Chibiny tundras. But due consideration should also be given to the geological and petrographical characters of these ores, without attempting to make these particular solutions universally applicable to the study of the Chibiny rocks.

S. M. Kirov Kola Base,
Academy of Sciences of the USSR.

Received
15. II. 1944.

REFERENCES

[1] B. N. Melentiev, C. R. Acad. Sci. URSS, XXV (1939); XXXIX (1943).
[2] B. N. Melentiev, ibid., XXXVIII, No. 2—3 (1943).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 9

GEOLOGY

THE PROVINCE OF SCHEELITE-BEARING SKARNS IN CENTRAL ASIA

By H. M. ABDULLAEV

(Communicated by A. E. Fersman, Member of the Academy, 13. VII. 1944)

Formation of Scheelite-Bearing Skarns as Dependent on Geological Peculiarities of Central Asia

Central Asia is a well-pronounced province of scheelite-bearing skarns. The latter are confined to the Palaeozoic rocks of the folded systems which, by analogy with the Urals, are termed by A. E. Fersman the «uralides» (Western Tian-Shan mountain systems). He juxtaposed these systems to the mountain systems of the Pamir, the «caucasides», in which no scheelite-bearing skarns have as yet been discovered.

The scheelite-bearing skarns are concentrated within the «northern» and «central arcs» of Central Asia (after D. V. Nalivkin), while in the «southern arcs» (Pamir) no scheelite-bearing skarns have been found.

The highest concentration of scheelite-bearing skarns has been recorded in the following mountain systems of Central Asia (from north to south):

(a) in the mountain systems in the region of Lake Son-Kul (Naryn subzone of the middle system);

(b) in the mountain system of Mogol-tau and Kurama (Kurama subzone of the middle zone);

(c) in the mountain systems of Nur-Ata, Zerabulak and Kara-Tyube (Nur-Ata and partly Zeravshan subzones of the southern zone);

(d) in the Zeravshan-Alai mountain system (Zeravshan and partly Alai subzone of the southern zone);

(e) in the Ghissar mountain system (Ghissar subzone of the south-marginal zone).

The Ghissar system is the extreme southern limit of distribution of the scheelite-bearing skarns. Further on the mountain systems—the «uralides» of Central Asia—are replaced by the «pamirides» which stand apart from the «uralides» both in geological structure and in the degree to which they are ore-bearing.

The scheelite-bearing skarns are located in the areas of Variscian anticlinal elevations of Central Asia and are not infrequently confined to small folds and limbs of these anticlinoria; this is a regional regularity. The Zeravshan, Nur-Ata, Ghissar and Alai mountain systems are regions of great Variscian anticlinoria, complicated by minor folds.

The scheelite-bearing skarns of Central Asia are associated with granodiorites and quartz dykes. V. A. Nikolaev, V. I. Popov, H. M. Abdullaev and others have pointed out the extensive development of rocks of this series in the Nur-Ata, Zeravshan, Ghissar and Alai mountain systems, and in the region of Lake Son-Kul. In the Kirghiz, Talass and Turkestan ranges, *i. e.* in those where scheelite-bearing skarns are poorly developed, if at all, granites predominate over granodiorites.

In the region of development of granitic intrusions proper, hardly any scheelite-bearing skarns are found, and tungsten is concentrated in quartz veins and wolframite.

The scheelite-bearing skarns are genetically related to the abyssal intrusions of the Variscian time. Such are the intrusive massifs of the Nur-Ata, Zerabulak, Kara-Tyube mountains, the Zeravshan and Ghissar ranges.

The intrusions with which scheelite-bearing skarns are associated are very rich in vein rocks. There is not a single skarn field without vein rocks of aplite, pegmatite or others. The age relations between the vein rocks and the skarns vary as to the skarn field. In some of them the skarns are younger than the vein rocks, in others they are older (Kumbel, Rarz).

The intrusive bodies with which the scheelite-bearing skarns are associated, are in most cases stretched along the strike of folds, being represented by interformational bodies. They are not infrequently of a regular rounded shape, about 1 km in cross-section (Rarz, Meliksu, Besh-Archa, *etc.*). In this particular case we probably have to do with summits of dome-shaped satellites. The bulk of the scheelite-bearing skarns of Central Asia are located near these small satellites (Meliksu, Besh-Archa, *etc.*).

In some deposits the intrusive massifs with which the scheelite-bearing skarns are associated, have been subject to processes of autoskarnification (Chorukh-Dairon, Liangar, Kumbel, *etc.*) accompanied with the formation of skarns carrying scheelite. The process of autoscarnification undoubtedly followed the consolidation of the apical part of the intrusive; consequently, the formation of skarns took place after the contact phase proper. The concentration of gases and solutions followed the consolidation of the apical part of the intrusive, which plugged the way of their exit. They were rising along the contact, the non-homogeneous medium of which was the less resistant zone at the period of cooling of the intrusive. This accounts for the fact that over 80 per cent of the scheelite-bearing skarns of Central Asia are located directly on contacts. In those cases where the gases and solutions were rising along fissures formed in the consolidated part of the intrusive, a process of autoskarnification was operative.

In the areas of development of scheelite-bearing skarns an extensive manifestation of thermal metamorphism with the formation of marbles and of different hornstones is observed. It is to this fact that the formation of coarse-grained marbles was due,—of the «prepared calcite» which is easily subject to metasomatic replacement in the process of «skarn-formation». Thus, the manifestation of thermal metamorphism is the first stage in the formation of skarn deposits.

Some Peculiar Features of the Scheelite-Bearing Skarns of Central Asia

The skarns of Central Asia are formed by way of replacement of rocks of the most diverse composition. Nearly all the minerals and concentrations of scheelite are formed under conditions of a strong metasomatosis, with a supply and removal of a very large amount of chemical components. In the process of metasomatosis in the carbonate medium, especially in a later stage of development, residual calcite is of essential importance. This calcite becomes an important factor in mineral formation, the shapes, sizes, and quantities of the minerals formed in the later phases of ore-formation being largely dependent on the shape, size and quantity of residual calcite.

Among the rocks subject to skarn-forming processes, the following may be mentioned: (1) limestones, (2) shales, (3) sandstones and conglomerates, (4) granodiorites and adamellites. It is in the limestones that this process obtained its widest development.

With respect to the active contact, scheelite-bearing skarns may occur:

(a) in the zone of direct contact, this group of deposits prevailing and constituting about 80 per cent of all the deposits recorded; to this group belong the Liangar, Kabuty, Ingichke and other deposits;

(b) at a certain distance from the contact, within the roof, being confined to the interlayer exfoliation or cutting and intersecting fissures—Kukhi-safit, Koy-tash, *etc.*;

(c) in the rocks of the parent intrusive as a result of autoskarnification—Choruck-Dairon, Chalai, *etc.*

In the scheelite deposits of Central Asia the following morphological types of skarns may be distinguished:

(a) Contact deposits, which are the most widespread, making up about 50 per cent of all the known skarn-scheelite bodies of Central Asia. They occur at the contact of intrusive bodies with the rocks of the roof and have the shape of lenses, contact veins, pockets or ore-shoots, controlled by the surface of the contact. The distribution of skarns in the zone of contact is usually a non-uniform and discontinuous one; their thickness is no constant one, either. As a rule, they occur conformably to the contact. An instance of this morphological type is furnished by the Ingichke deposit, the skarns of which are developed in periclinal pitches in areas of curved contacts and domes.

(b) Bed-like bodies in the rocks of the roof are associated with exfoliations and selective metasomatosis of individual beds. Exfoliations are most readily produced at the contact of heterogeneous beds with dissimilar physical properties. In the shaping of bed-like bodies the screening rôle of shale rocks with a low water-permeability is of no small significance (Malkhura, Koy-tash).

(c) Skarn bodies in fissures and fractures are rather scarce and are developed both in the rocks of the roof (Kumbel, Meliksu), and in the parent massif (Chorukh-Dairon).

At the scheelite deposits of Central Asia two types of skarns are formed which differ sharply in time of formation, mineralogical composition and morphology: dark silicates (tactites) and light silicates (calcareo-silicate hornstones).

The scheelite and other ore-minerals are restricted solely to the tactite varieties of skarns.

The zonal distribution of light silicates, tactites and aplitic granites, pointed out by Hess, Larsen and other geologists, has not been confirmed.

The tactites have a compound mineralogical composition. The predominant rock-forming minerals are pyroxenes (diopside-hedenbergite), garnets (andradite-grossularite), amphiboles (hornblende, actinolite, uralite), vesuvianite, scapolite, epidote-zoisite, quartz, calcite; of the ore-minerals, pyrrhotine, magnetite, scheelite (see table).

According to the prevalence of a definite mineral the following mineralogical types of deposits of scheelite-bearing skarns may be distinguished:

1) with a prevalence of pyroxene (Liangar, Ingichke)
2) » » » » garnet (Kumbel)
3) » » » » pyrrhotine (Meliksu, Kukhi-safit)
4) » » » » magnetite (Chalai).

In some of the deposits a definite dependence of the mineralogical composition of the skarns upon the composition of the enclosing rocks is noticeable. In the Ingichke deposit the skarns are formed solely at the expense of limestones and occur at the contact with granodiorites, having largely a pyroxenic composition, poor in alumina. At Liangar, in the Central area, the skarns are also formed at the expense of limestones and consist mainly of pyroxene. In the southern part, according as the shales make their appearance, garnet becomes prevalent in the composition of the skarns.

The skarns of the Kalkan-ata deposit, formed at the expense of shales, are composed exclusively of garnet. In the Chorukh-Dairon deposit the replacement of granodiorite is accompanied with scapolite and garnet formations. At Liangar and Kumbel the skarns, formed at the expense of adamellite, consist of epidote and garnet.

In the formation of skarn minerals a very important part is played by residual calcite, as well as by fissures originating at the period of mineralization. Nearly all of the silicates, as well as scheelite, magnetite, pyrrhotine and even

quartz develop by way of a replacement of calcite. The latter also facilitates the development of crystallographical faces of the minerals.

The distribution of scheelite in the skarn is a most variable one:

1) a uniform distribution throughout the mass of the skarns (Ingichke, Meliksu);

2) as pockets and veinlets (Liangar);

3) as veins and veinlets cutting the skarns (Kumbel).

A definite relation is observed between the coarseness of the grains of skarn minerals and of scheelite, *e. g.*, at Liangar both the skarn minerals and the schee-

Minerals of Scheelite-Bearing Skarns

Minerals	Distribution		
	widespread	medium frequency	scarce
I. Native			gold, bismuth, sulphur*
II. Sulphides	pyrrhotine, pyrite, chalcopyrite	molybdenite, galena, bismuthinite, sphalerite, arsenopyrite	wolframite, arsenopyrite, tetrahedrite, tennantite, antimonite, cinnabar, stannite, chalcosite*, covellite*, bornite, marcasite
III. Oxides	quartz, limonite	hematite, magnetite, tungstite*, opal, psilomelane*	cassiterite, cuprite*
IV. Carbonates	calcite	malachite*, azurite*, aragonite	siderite, cerussite*, ankerite*, smithsonite*
V. Silicates	pyroxene, garnet, epidote-zoisite	vesuvianite, scapolite, wollastonite, plagioclase, orthoclase, actinolite, tremolite, muscovite, sericite, biotite, phlogopite, chlorites, zircon	rhodonite, ilvaite, vermiculite, serpentine, asbestos, chrysocolla, prehnite, orthite
VI. Titanium silicates		titanite	
VII. Phosphates and borates		apatite, tourmaline	
VIII. Halogen compounds			fluorite
IX. Tungstates and molybdates	scheelite	povellite*	

* Supergene

lite are coarse-grained, whereas at Ingichke, Rarz and other deposits, both are fine-grained.

Of decisive importance in the localization of scheelite are the following factors:

(a) the permeability of skarns to tungsten-bearing solutions. The permeability of skarns may be due to their deformation, either in the form of reduced strength of the interstices of the minerals, or of small cracks and zones of crushing;

(b) the presence in the skarns of residual calcite. The size and distribution of the latter are of essential importance in the localization of scheelite, since the readily soluble and highly active calcite reacts and gives up calcium for the building up of scheelite.

Stages and Phases in the Formation of the Scheelite-Bearing Skarns of Central Asia

The formation of scheelite-bearing skarns is a part of the whole process of development of the geological history of individual ore fields, and they cannot be viewed separately. In this process the following stages and phases may be distinguished:

I stage. The initial stage of the process is characterized by an extensive display of orogenetic movements, owing to which a structure favourable for the subsequent development of the ore field is produced, and by intrusion.

II stage. The phenomenon of intrusion is accompanied by an extensive display of thermal effect upon the rocks of the roof with their recrystallization and marmorization. To this stage belong also endocontact alterations as a result of an assimilation of rocks by the intruded magma, with an enrichment of the latter in CaO. The formation of calcareo-silicate hornstones refers to this type.

III stage. Skarn-formation. As a result of the formation of a crust of the intrusive and the plugging of the outlets a concentration of emanations takes place. Through the contacts between the intrusive and the rock of the roof, and the cracks in the already consolidated part of the massif, the emanations rise and metasomatically replace the rocks of the roof or of the mother rocks itself, under the formation of skarns. The physical state of the emanation seems to be a gaseous-aqueous one, the gaseous phase prevailing at the beginning, and the aqueous, at the end of the stage.

This stage is the most decisive and complicated one and consists of several phases.

In the first, s i l i c a t e, phase calcareo-ferruginous silicates are formed: pyroxene, garnet, vesuvianite, *etc.*, their formation taking place under conditions of a strong metasomatosis, the distribution of these minerals and their sizes being controlled also by the residual calcite which at a later phase was also replaced by ore-minerals.

The first phase of silicate formation is followed by a second one, a q u a r t z-o r e phase, which may succeed the first without any special break or with a sharp break.

The deposition of scheelite and other ore-minerals in the skarns is not occasional, but is the result of a prolonged and complicated process. It is impossible to view the formation of ore-minerals separately from the formation of skarns, the whole process of mineralization should be considered as a whole.

Geological Institute.
Academy of Sciences of the Uzbeck SSR.

Received
13. VII. 1944.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 9

GEOLOGY

ON THE FORMATION OF UNDERGROUND WATERS OF THE DNIEPER-DONETZ DEPRESSION

By K. I. MAKOV

(Communicated by F. P. Savarensky, Member of the Academy, 29. II. 1944)

The chemical composition of the underground waters of the Dnieper-Donetz depression has become what it is now after a very long period of time. The principal agencies which have determined the chemistry of the waters from different horizons of the Dnieper-Donetz depression may have been the following: (a) the facies of water-bearing strata; (b) the chemical and mineralogical composition of the water-bearing formations;(c) the tectonics of the region, in the first place epeirogenesis.

The facies of water-bearing strata determines the initial composition of the underground waters confined to undrained formations (underground waters *in situ*); these are waters which formed at the same time with (or shortly upon) the water-bearing rock, and later on became metamorphosed to some degree or other.

The chemical and mineralogical composition of the water-bearing formations and of the rocks which are in contact with the water-bearing strata (bottom, roof), under the conditions of the Dnieper-Donetz depression was essential and continues to be so for the formation of the chemical composition of those underground waters only which are circulating in carbonate formations; it is of lesser importance for the waters circulating in quartz sands, the latter being practically insoluble. Numerous analytical data show that, other conditions being equal, within the Dnieper-Donetz depression such waters are confined to quartz sands whose mineralization is considerably less than that of the waters circulating in carbonate formations.

In the case where the water-bearing formation and the surrounding rocks are rich in readily soluble rocks, the degree of mineralization of the waters is the highest (the waters of the Carboniferous from the regions of Romny, of Starobelsk, the waters of the «variegated» rocks, of the deep horizons of the Devonian, the waters of the Jurassic, contaminated with H_2S, those of the Cenomanian, *etc.*). Thus, it is the facies of the water-bearing deposits and the chemical and mineralogical composition of the water-bearing formations that have determined the original material (components) that was responsible for and continues to influence the chemical composition of the underground waters of the depression.

The epeirogenetic movements involving vast areas of the Dnieper-Donetz depression played the chief part (as compared to folding) in the further evolution of the underground waters.

The folding is responsible, first, for the isolation of the separate areas of development of aquifers or, on the contrary, for their fusing; secondly, at some places it brought the younger water-bearing deposits in contact with the deep-lying ancient undrained rock formations. Either of these agencies might have an important effect upon the process of formation of underground waters in those

regions where folding was operative. However, within the depression rather small regions were affected by folding which was mainly completed by the middle of the Mesozoic.

The epeirogenetic movements which proceeded throughout the whole geological history brought about: (a) the distribution and shifting of the catchment areas (areas of supply) and zones of drainage; (b) the formation of circulation zones; (c) the opening and closing of areas for the run-off of underground waters.

The epeirogenetic movements caused the formation of dry land with a varying development of the hydrographic network. During the positive phases of epeirogenesis within the depression (predominating in time over the negative epeirogenesis) in the area of this dry land an extensive freshening of aquifers took place [the replacement of highly mineralized (sea) waters by fresh (atmospheric) waters].

The accumulation of fresh waters proceeded to a large scale at the periods of breaks in the deposition of sediments—during the continental breaks (in the north-western part, a prolonged continental period after the Devonian to the Upper Jurassic; in the east of the depression, the period after the Carboniferous up to the deposition of the Cenomanian; in the central part of the depression, the continental period from the end of the Carboniferous to the Middle Jurassic; over the greater part of the depression, the Middle Jurassic time and the time up to the Cenomanian transgression, a short break at the end of the Cenomanian; persisting continental conditions in the north-eastern part, beginning with the end of the Cretaceous up to the present time, *etc.*).

In the geological past the process of fresh water formation over the newly formed dry land proceeded according to the following scheme: the fresh atmospheric waters supplied at the divides penetrated to the aquifer saturated with sea water, and produced a difference of pressures, thus displacing the highly mineralized waters to the lower parts of the relief (ravines, river valleys). This process continued throughout the period during which this portion of land emerged above sea level. At the same time an intense weathering was active, producing an increasing friability in loose rocks and hence an increased percolation, and the formation of cracks in solid rocks. In the carbonate rocks of the Devonian, Carboniferous and Cretaceous the formation of cracks was accompanied (and followed) by an intense formation of sink-hole; owing to this process, extensive cavities were formed in which later on great reserves of underground waters accumulated.

At the same time with the offset of highly mineralized waters within the areas of supply and in the zone of drainage a slow secular shifting went on and goes on still (according to the rate of movement which in the long run was also determined by the range of the vertical shifting of the continent) of fresh waters from the areas of supply towards the zone of circulation, and from this, towards the areas of run-off.

An analysis of the materials on the geology and hydrogeology of the region shows that the areas of supply of aquifers throughout the geological history of the depression were distributed as follows:

(a) At the periods of active positive epeirogenesis, over the greater part of the area of the depression (the north-eastern part, the central part, the south-western part). Thus, *e. g.*, at the end of the Cenomanian time the feeding of the aquifer in Cenomanian sands proceeded over the whole area of development; the processes of feeding of the marlaceous-cretaceous aquifer in the north-western part of the depression in the post-Cretaceous time were extensively developed, *etc.*

(b) At the periods of slight epeirogenetic upheavals the feeding of aquifers proceeded within some isolated areas. Thus, *e. g.*, in the Lower Cretaceous time the feeding of the Jurassic horizon proceeded in the central part of the depression and, subsequently, in the north-eastern part of the depression; the feeding of the Buchak-Konev horizon in the recent time, in the north-western and south-western parts of the depression, *etc.*

Within the areas where negative epeirogenesis was at play, a very slow movement (dying-out of movement) of underground waters was going on towards the

areas of discharge (areas with the least marks of piezometers of waters and with the least marks of the roof of the water-bearing rocks), or where there was no areas of run-off in the regions of subsidences, the movement of underground waters ceased altogether, and a basin of pressure underground waters was formed and existed for a certain time, during which the processes of rejuvenation of these waters ceased. This continued up to the time when the next positive phase of epeirogenesis would produce erosion and exposure of these aquifers and thereby secure either a local drainage or a general run-off underground waters from the previously closed aquifer.

In nearly all the large hydrogeological provinces the regularities here described are found in various modifications to hold for the accumulation of underground waters (East-Russian depression, depression adjoining the Black Sea, the Cis-Carpathian depression, the Main Devonian field of the Russian platform, *etc.*).

Within the area of the Dnieper-Donetz depression the absence of discharge accounts for the occurrence of underground waters in deep-lying strata of the Devonian, Carboniferous and Permian of the central part of the depression, a strong mineralization of the Buchak waters in some parts of the Poltava region, where the wash-out of the Dnieper has not touched the Buchak deposits (east of the meridian of Kremenchug), *etc.*

In this way the mineralization of the deep underground waters of the depression is related to the epeirogenetic movements which determined the palaeogeographical environment of the past ages. Hence, in order to elucidate the problems of the formation of the underground waters of provinces similar to the Dnieper-Donetz depression, it is necessary in the first place to study in detail the geological history of the region. The accumulation and generalization of the facts concerning the formation of the underground waters of the region throughout its geological history will make it possible to speak of the formation of underground waters in the geological past, of the geological history of the underground waters.

Institute of Geological Sciences.
Academy of Sciences of the USSR.

Received
29. II. 1944.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 9

CYTOLOGY

ZYMONUCLEIC ACID IN CELLS OF A NORMAL REGENERATE AND IN THOSE OF MALIGNANT GROWTH

By G. ROSKIN and G. KHARLOVA

(Communicated by I. I. Schmalhausen, Member of the Academy, 4. II. 1944)

Brachet ([1], [2]) has shown that egg-cells of amphibians are rich in cytoplasmic nucleic acid (most probably this is the zymonucleic acid, or some acid closely approaching to the latter) which is being consumed in the process of embryogenesis. It has been demonstrated by the same writer that the basophily of the plasm of the egg-cell is accounted for by the presence of zymonucleic acid and that, according as its amount decreases, the basophily of the cells of the different tissues forming anew becomes less intense, too. Thus, an attempt was made to explain the varying basophily of a developing egg, as well as of cells in the process of histogenesis Having applied Brachet's method to the investigation of the histogenesis of the chick embryo, E. Hershman of this Laboratory was able fully to confirm the data reported by Brachet; moreover, on the basis of a special study of the heart musculature, Hershman showed that during the process of the formation of the myocardium, there is observed a decrease in the basophily of the plasm and a reduced content of zymonucleic acid in the latter.

In the present work an attempt is made to find out, whether the presence in the cells of cytoplasmic nucleic acid is associated with the form-building processes of cytogenesis and histogenesis, or with an ability of intense growth and division only. We thought it of interest, moreover, to find out whether a cell which has consumed during development its store of zymonucleic acid may be able to restore it in its plasm. As material for our work served: (1) bits of regenerating muscular and cartilagenous tissue of axolotl; (2) various malignant growths of human patients. For determining the cytoplasmic nucleic acid we had recourse to the method of Brachet: the sections were treated with ribonuclease, with subsequent staining with methylgreen-pyronine according to Unna, or by the method of Dominici. In the former case, those of the cell components which contained zymonucleic acid stained with pyronine an intense red (the effect vanished if the cells had previously been treated with ribonuclease).

Regeneration was studied on axolotl limbs, part of which were severed and served as control preparation, while the regenerating areas were fixed at various intervals with 96° alcohol. Observations carried out on the tissue of striated muscles showed that the less differentiated the cell, the more intensely does its plasm stain with pyronine. In cells of myoblastic type, whose structure is scarcely detectable if at all, areas of an especially intense colour are sometimes found to surround the nucleus. From experiments with ribonuclease it is evident that in this case basophily is dependent on the appearance in the cells of plasmic nucleic acid. In the regenerating areas of the cartilagenous tissue a sharply pronounced basophilic plasm of the cells of the perichondrium and a highly basophilic one of the carti-

lage cells is observable. The most basophilic plasm is that encountered in the peripheral cells of the cartilage; that of the cells situated in the centre is less basophilic. The basophilic substance does not permeate the plasm of the cells homogeneously, but fills the cytoplasm as a finely granular matter. Experiments with ribonuclease (control tests with hot distilled water) have shown the sharply pronounced basophily of the cells of the perichondrium and of the young cartilagenous cells to be due to the plasmic nucleic acid present in them.

Let us now compare the observations made on the cells of regenerate with the results of our study of cells of malignant growth in man. We have examined a number of tumours differing as to their genesis: cancer of the mammary gland, of the lip, of the stomach, basilioma of the cheek, cancer of the tongue, *etc.* Let us consider as an example the cancer of the stomach whose cells are rather coarse, and can therefore easily be examined. The basophily of the plasm of cancerous cells is clearly borne out on preparations, but the degree of expression is not the same in various cells. A general rule may be laid down: the farther the cells are from the centre of multiplication, *i.e.* from the regions rich in mitoses, the paler is the colour of the plasm of the cells when stained with pyronine (in case Unna's method is applied). Alongside of cells whose cytoplasm is thoroughly impregnated with the basophilic component, such cells also occur in which the basophilic substance is arranged along the periphery, or also around the nucleus only. The experiments with ribonuclease show that the greatest part of the basophilic component vanishes under the effect of this enzyme. In the plasm of the cancerous cells of a stomach malignant growth two basophilic components are observed, one of which and the most important at that, is the cytoplasmic nucleic acid. This component of the plasm is the most intensely pronounced in those parts of the tumour where the cells are in the state of intense division.

Our observations on cells of cancerous growth of the lip revealed most peculiar alterations which could also be observed in other tumours in human patients. The large size of the cells of this kind of tumours permitted to examine the distribution of the basophilic substance in detail. When staining after Unna is applied, the pyroninophilic substance is seen to be distributed ununiformly over the different parts and cells of the tumour. Most of the cells take on a light-pink colour which is uniformly distributed throughout the protoplasm. These are mainly the strongly hypertrophied cells whose nuclei are very large and bear traces of degeneration. Less hypertrophied cells containing nuclei of about a normal structure, though poor in chromatine, show separate areas rich in basophilic plasm: these areas may be situated at the periphery of the cell, or perinuclearly. Unlike this, in cells whose nuclei are rich in chromatine, the cytoplasm is intensely impregnated with the basophilic substance; these cells stain a deep red. From these observations it may be inferred that the plasm of the cancerous cells examined by us contains two basophilic components at least. One of them, by which the plasm is impregnated in a diffuse and uniform way, is responsible for the weak basophily and vanishes as soon as the material is treated with hot distilled water; it is met as a rule in any cancerous cell. Another basophilic component of the plasm is responsible for the intense red colour taken on by the whole cytoplasm or by its definite portion in the case of pyronine staining. This component occurs in large amounts in moderately hypertrophied cells, with a more or less intact nuclear structure, *i. e.* in such cells as seem to be capable of a further intense growth and proliferation. It should be noted in this place that an impoverishment of the nuclei with chromatine is associated with a reduction of this basophilic substance. The second basophilic component remains unaltered when the preparations are treated with hot distilled water, whereas it vanished altogether in those treated with ribonuclease. Thus, in intensely proliferating cancerous cells we find plasmic nucleic acid, the amount of which decreases along with the hypertrophy of the cells and may be utterly lacking in the so-called giant cells.

All what has been said above permits of the following conclusions:

1) Like embryonic cells, the cells of a regenerating muscular or cartilagenous tissue contain cytoplasmic nucleic acid of the type of zymonucleic acid.

2) In the course of differentiation, the content of zymonucleic acid in the cells of regenerating tissues becomes lower.

3) As a rule, the cells of cancerous growth contain in their plasm a considerable amount of zymonucleic acid.

4) It may be assumed that the highest in cytoplasmic nucleic acids are cancerous cells in the state of intense division.

5) Such cancerous cells as are increasing in size, hypertrophied and the so-called giant cells, contain very little zymonucleic acid, if at all.

6) All the observations referred to above make it evident that the cells which have lost their cytoplasmic nucleic acid in the process of embryogenesis and histogenesis, under conditions of the regeneration process or of malignant growth may form their cytoplasmic nucleic acid anew and thus approach to non-differentiated embryonic cells.

7) The presence of cytoplasmic nucleic acid is not associated with formative processes, as it might be inferred from the study of embryonic cells and processes of histogenesis, but rather with processes of intense division. This is evidenced by observations carried out on cells of malignant growths, where no phenomena of cell differentiation take place.

8) Brachet (1940—1941) showed that the plasm of the secretory cells (pancreas, liver, stomachal glands), *i. e.* of such cells as are producing definite products, contain moreover plasmic pentosonucleic acid. This component s h o u l d n o t b e i n t e r p r e t e d a s a s p e c i f i c a g e n t o f c e l l d i v i s i o n, but its presence is rather to be associated w i t h a n u m b e r o f s y n t h e t i c p r o c e s s e s which take place in the cell, and whose complex may under certain conditions lead to cell division.

Department of Histology.
M. V. Lomonossoff State University.
Moscow.

Received
4. II. 1944.

REFERENCES

[1] B r a c h e t, C. R. Soc biol., 133 (1940). [2] B r a c h e t, Enzymologia, 10 (1941).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 9

PLANT PHYSIOLOGY

RUBBER FORMATION IN ROOTS OF KOK-SAGHYZ DUG UNDER CONDITIONS OF IMPEDED GAS EXCHANGE

By O. J. SOBOLEVSKAYA

(Communicated by A. A. Richter, Member of the Academy, 9. VI. 1944)

Early in August 1943 T. D. Lyssenko, Member of the Academy, sent to the analytical-testing laboratory of the Institute of Rubber Plants several root samples, to have rubber content determined in them at the day they were received and after five days of storage; the conditions of storage have not been specified.

Having taken interest in the question of how the content of rubber may be affected by storage, we undertook a set of experiments based on the literary data available.

The mechanism of rubber formation in living plants is not quite clear yet; various writers have furnished evidence, however, marking the separate steps which bring us nearer to the understanding of rubber synthesis.

1. The synthesis of rubber in root rubber-plants takes place in the laticiferous tubules of the root ([2–4, 9]).

2. The formation of rubber by plants is in no direct relation to photosynthesis ([1, 4, 9]).

3. The accumulation of rubber in the roots of a living plant proceeds most actively by the end of the growing period, after the formation of the plant itself has been achieved ([5]).

Starting from these statements it may be supposed that, the conditions being favourable, rubber synthesis may proceed in the roots of live plants even in those cases where the latter have been dug out of the ground. Similar facts have actually been reported ([7]).

The following considerations have been laid at the basis of our experiments:

1. Rubber hydrocarbon is a compound of a reducing character; its synthesis is therefore likely to proceed under conditions of a low oxygen content in the plant tissues and in the surrounding medium.

2. In order that rubber synthesis take place in dug roots, the latter should be kept alive; at the same time the conditions of storage should be such as to prevent any growth process.

The roots taken for experiment were made equal in length and each root was halved along the longitudinal axis. One half was taken into experiment, another kept for control.

The roots of each experimental sample were tightly wrapped into wax-paper in order to prevent them from loosing moisture rapidly, and put into a box; an atmosphere rich in products of gas exchange was in this way created within each lot of roots.

After five days the experiment was interrupted; the roots were fixed with steam, and rubber content estimated in them after the alkali method ([10]).

The loss in dry weight of root on respiration during experiment was allowed for when calculating the percentage of rubber.

Another experimental run comprised samples of roots which had preliminarily been cut into small pieces about 1 cm long.

Under experiment were roots of various age and different dates of digging. The results obtained are summarized in Table 1.

Table 1

Rubber Production in Roots of Kok-saghyz under Conditions of Impeded Gas Exchange

Age and origin of the plants	No. of sample	Experimental run	Date of digging (1943)	Mean weight of raw root in g	Weight of raw root per sample in g	Moisture content in %	Rubber content in 100 plants: before experiment (in g / in %)	Rubber content in 100 plants: after experiment (in g / in %)	Increment of rubber to initial weight in g / in %
First-year plants, sown in spring 1943	1	Entire roots	11.X	8.3	460	68.4	30.8	33.8	2.9 / 9.7
Plants one-year-old, cuttings planted in fall 1942	2	» »	3.VIII	27	430	68.3	54.9 / 5.7	61.8 / 7.0	6.9 / 12.4
	3	» »	28.VIII	50	480	73.9	84.3 / 6.7	122.2 / 9.7	37.9 / 44.9
	4	» »	11.X	41.6	229	76.5	78.4 / 8.0	93.3 / 9.5	14.9* / 18.7
Plants three-year-old, sown in spring 1943	5	» »	3.VIII	19.3	382	60.4	34.1 / 6.0	48.3 / 8.5	14.3* / 41.8
	6	» »	28.VIII	8.9	455	76.5	92.4 / 7.1	119.6 / 9.2	27.2 / 29.3
First-year plants, sown in spring 1943	7	Roots cut	11.X	—	500	68.4	9.2	9.0**	—
Plants one-year-old, cuttings planted in fall 1942	8	» »	3.VIII	—	405	64.3	13.3	13.4	—
			3.VIII	—	410	68.1	13.9	14.0	—

* Loss of dry matter on respiration has not been determined in this run.

** Rubber content in cut roots calculated per 500 g of raw material, the quantity of roots being not evaluated in these samples.

In all experimental samples involving entire roots kept under conditions of inhibited gas exchange the content of rubber has been found to have increased as compared with the control; unlike this, the samples made up of cut roots showed no change in their rubber content, after allowance for respiration had been made. The cause of this phenomenon will be discussed in the subsequent.

The increase in rubber content varies as to the root sample within the same experiment. This difference in the efficiency of the experiment should evidently be put down to the biological condition of the experimental plants.

The store of carbohydrates, in particular of inuline, and the filling of the laticiferous ducts with rubber, seem to limit the process of rubber formation in the roots dug at various periods of rubber accumulation by a growing plant.

The results of analyses of roots dug at various dates (samples 2, 3, 4), as well as literary data (4) justify such an assumption.

The lack of rubber increment in the experiment with cut roots may be accounted for by the fact that when cut the root-sections are enriched with air conside-

rably; thus, when they get under experiment, they are rich in oxygen. Owing to this, the experimental conditions of respiration differ from those in the experiment on the entire roots. The figures of loss in dry weight go to confirm this supposition: the consumption of dry matter in the process of respiration in the course of experiments with entire roots makes up about 6—8 per cent to the initial amount, while in the experiments with cut roots it amounts to 14—16 per cent.

It may be assumed that the supply of roots with oxygen is guaranteed not only by its being present in the soil medium, within the zone of root distribution, but by way of the life-active assimilatory organs which are in the state of an intense gas exchange with the air medium, and even produce free oxygen in the tissues in the process of photosynthesis.

If this is so, the arrest of photosynthesis, for example, the loss of life-active leaves, should affect the gas exchange in the root, leading to a decrease in the redox potential of the tissues. Thus, during their period of summer quiescence, kok-saghyz plants increase in rubber by about 25 per cent (6).

This seems also to account for the increased rubber content in the roots of krym-saghyz in the experiments by D. M. Axelrod (Table 2).

Table 2

The Pre-harvesting Enrichment of Krym-saghyz Roots by Means of Covering the Rosettes with Soil, Previous to Root Digging (experiments by D. M. Axelrod, Atabaevo, st. Turkestan, 1942)

Material	Experimental run	Rubber content in%		Rubber increment in % to control
		in control	in exper.	
Culture 18-month-old	Rosettes covered with soil 15 days before harvesting	4.02	5.79	44.3
Culture two-year-old	Same, 10 days before harvesting	5.81	6.51	

What has been said above makes us believe that the formation of rubber in the roots of rubber plants, which takes place independently of photosynthesis, is but a particular case of production of reduced compounds associated with the decay of reserve hydrocarbons in the process of intramolecular respiration.

This assumption is confirmed by the very fact of rubber formation in the laticiferous system of the root which is an organ where the total energetic material of the plant is stored.

Hence it is clear why the most intense accumulation of rubber by a normally growing plant proceeds towards the end of the growing period, when the assimilation capacity of the plant is abated considerably, and may even be fully stopped. Having the longest period of rest, the tau-saghyz is at the same time rich in rubber.

As a result of the investigations carried out, it may be regarded as firmly established that the synthesis of rubber in dug kok-saghyz roots during short storage periods proceeds under conditions of inhibited gas exchange, the content of oxygen both in the tissues of the plant and in the surrounding medium being a rather low one.

The accumulation of rubber may occasionally attain rather high figures, according to the biological condition of the plant.

As the accumulation of rubber by the roots may be limited by the filling of the laticiferous ducts with rubber, it is necessary in the subsequent to test the

efficiency of storage of roots with a preliminarily opened latex system (cutting-off rosettes).

In order to increase the rubber content in the raw material under industrial conditions, it is necessary to investigate the gas regime in the lot and to find out the optimal concentrations of oxygen and hydrocarbon, of moisture content and temperature for the rubber production in the case of lot storage of the roots.

In concluding I wish to express my deep gratitude to T. D. Lyssenko, Member of the Academy, who suggested the idea of the experiments described in the present note.

Received
31. I. 1944.

REFERENCES

[1] И. И. Блохинцева, Биохимия и физиология каучуконосных растений, сборн. II, М., 1939. [2] И. И. Блохинцева, Ботан. журн., № 4 (1939). [3] I. I. Blokhintzeva, Bull. Acad. Sci. URSS, sér. biol., No. 43 (1940). [4] Ф. П. Мазанко, Биохимия и физиология каучуконосных растений, сборн. II, М., 1939. [5] М. С. Маштаков, Вестн. техн. культур, № 3 (1940). [6] S. M. Mashtakov, C. R. Acad. Sci. URSS, XIX, No. 4 (1938). [7] Г. Б. Нейман и Н. Добровольская, Доклады ВАСХНИЛ, вып. 22 (1940). [8] А. А. Ничипорович, Промышленные каучуконосы СССР, изд. 2, М., 1938. [9] A. A. Prokofiev, Bull. Acad. Sci. URSS, sér. biol., No. 6 (1939). [10] О. Ю. Соболевская, Инструкция по учету динамики каучуконакопления и определения каучука в корнях кок-сагыза, под ред. И. С. Половенко, М., 1943.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLIV, № 9

PHYSIOLOGIE

LES SPERMATOXINES ET LE PROBLÈME DE LA RÉGULATION ARTIFICIELLE DU SEXE CHEZ LES MAMMIFÈRES

Par V. SCHROEUDER

(Présenté par L. A. Orbeli, de l'Académie, le 24. XII. 1943)

Une longue série d'expériences de l'auteur ([1–7]) sur le problème de la régulation artificielle du sexe chez les mammifères et sur l'étude physiologique des spermatozoïdes des mammifères a montré que les spermatozoïdes contenant un X ou un Y chromosome revèlent un mouvement différent au champ électrique: grâce à leur nature biochimique différente les spermatozoïdes aux chromosomes X arrivent à l'anode, tandis que les spermatozoïdes aux chromosomes Y arrivent sur la cathode (méthode électrophorétique).

C'est la question d'application de cette méthode à la pratique de l'élevage et l'étude des cytoxines, surtout des spermatoxines, qui fait l'objet de la présente Note. L'étude des spermatoxines effectuée pendant plus de 40 ans par des auteurs différents a confirmé dans de nombreux travaux expérimentaux ([8–17]) les résultats de Metschnikoff et de Landsteiner à ce sujet.

Plusieurs auteurs ont étudié en detail la cellule du spermatozoïde, prise comme un antigène. Dans le travail de Heule et de Chambers a été constaté la présence d'antigènes différents dans la tête et dans la queue des spermatozoïdes ([18,19]).

L'étude des spermatoxines contribua au développement du vaste chapitre sur la stérilité de l'organisme des mâles et des femelles, qui ont été immunisés avec des spermatozoïdes ([20–29]).

C'est en rapport avec l'étude des cytotoxines, surtout des spermatoxines, qu'a été admise la possibilité d'obtenir un affaiblissement des spermatozoïdes avec X ou avec Y chromosomes dans l'organisme d'un mâle immunisé avec des faibles doses des spermatozoïdes X ou Y (les spermatozoïdes X ou Y reçus d'après notre méthode électrophorétique de l'anode ou de la cathode).

En cas d'un affaiblissement des spermatozoïdes avec X chromosomes, c'est la prédomination du sexe masculin qui a été attendue dans la postérité du mâle immunisé; en cas d'un affaiblissement des spermatozoïdes avec Y chromosomes ça devait être le sexe féminin.

1. Méthode expérimentale. Un lapin mâle, un bon producteur, a été immunisé à l'oreille avec des spermatozoïdes X ou Y, c'est-à-dire avec ceux de l'anode et ceux de la cathode reçus après l'électrophorèse pratiquée d'après notre méthode. Au bout de l'expérience les spermatozoïdes de l'anode (X spermatozoïdes) et ceux de la cathode (Y spermatozoïdes) one été mis séparément dans les éprouvettes stériles et resuspendus dans une solution stérile de Ring Z^4, pH=7.10.

Les lapins mâles ont été immunisés par les spermatozoïdes de l'anode (X) et par ceux de la cathode (Y).

L'immunisation durait dix jours avec des intervalles de 3—5 jours; les lapins recevaient pendant l'immunisation une dose de 1—4 cm³ de spermatozoïdes suspendus en Ring Z^d. Dans certaines séries d'expérience les lapins recevaient un nombre défini de spermatozoïdes contenus en 1 cm de la solution Ring Z^d.

Les mâles one été aussi immunisés dans certaines séries d'expériences par les antiséra spécifiques (X et Y). 9 jours après la fin de l'immunisation les mâles immunisés ont été croisés avec des femelles normales (non-immunisées); c'est dans leur descendance qu'on a étudié le nombre des mâles et des femelles (la corrélation des sexes).

Le croisement d'un mâle immunisé avec une femelle normale avait lieu pendant 10—15 jours après la fin de l'immunisation.

Dans les 4 séries d'expériences suivantes sont exposés les résultats obtenus sur l'immunisation des mâles-producteurs avec des doses différentes de spermatozoïdes X et Y ou avec des antiséra; c'est aussi le nombre des mâles et des femelles dans la descendance du lapin immunisé, qui est indiqué dans les 4 séries d'expériences suivantes.

2. Les résultats obtenus. I série. Immunisation des mâles-producteurs avec des spermatozoïdes X (obtenus de l'anode). Dans la descendance de ces males a été attendu la prédominance du sexe masculin en cas d'un dosage favorable.

Les résultats obtenus sont exposés dans la table 1.

Table 1

Immunisation des lapins avec des doses différentes d'X spermatozoïdes (anodiques)

Expériences №№	Lapins immunisés, №№ des mâles	Daet de l'immunisation et nombre de spermatozoïdes introduits		Nombre des descendants dans la postérité du ♂ immunisé et de la ♀ normale	Relation numérique des sexes dans la postérité	
					♀	♂
1	8	19.X 1940 2 22.X 2 25.X 2	6 cm³	6	2	4
2	16	18.X 0.5 20.X 1 25.X 1.5	3 cm³	28	10	18
3	3	1.XI 0.5 5.XI 1 11.XI 1.5	3 cm³	11	0	11
4	45	2.XI 0.5 7.XI 0.5 12.XI 1	2 cm³	5	0	5
5	54	2.XII 1 7.XII 1	2 cm³	7	0	7
6	5	3.XII 0.5 8.XII 0.5	1 cm³	12	3	9
7	20	4.XII 0.5 9.XII 0.5	1 cm³	14	2	12
8	89	5.XII 1 10.XII 1 12.XII 1	3 cm³= =31 200 cellules (spermatozoïdes)	8	0	8
				85 animaux	17.65%	82.35%

Table 2

Immunisation des lapins avec des doses différentes d'Y spermatozoïdes (cathodiques)

Expériences №№	Lapins immunisés, № des mâles	Date de l'immunisation et nombre des spermatozoïdes introduits	Nombre des descendants dans la postérité du ♂ immunisé et de la ♀ normale	Relation numérique des sexes dans la postérité ♀	♂
1	71	1.IX.1941 1.5 5.IX. 1.5 10.IX. 1.5 } 4.5 cm³ = 26.300 cellules (spermatozoïdes)	28	14 50%	14 50%
2	67	1.IX 0.5 5.IX 1 10.IX 1.5 } 3 cm³ = plus de 22000 cellules	6	2	4
3	14	1.IX 0.5 5.IX 0.5 10.IX 1 } 2 cm³ = 22000 cellules	17	6	11
4	25	5.IX 1 10.IX 1 12.IX 1 } 3 cm³ = 22000 cellules	5 28	2 35.70%	3 64.30%
5	70 et 72	2.IX 0.5 7.IX 1 12.IX 1 } 2.5 cm³ = 15500 cellules	76	35 46%	41 54%
6	63	2.IX 1 7.IX 1 } 2 cm³ = 9500 cellules	14	9	5
7	58	4.IX 0.5 9.IX 0.5 } 1 cm³ = 7200 cellules	20	16 73.53%	4 26.47%
		La moyenne du No. 7	20	16 80%	4 20%

Table 3

Immunisation avec des doses différentes de l'antisérum X

Expériences №№	Lapins immunisés, № des mâles	Date de l'immunisation et nombre des spermatozoïdes introduits	Nombre des descendants	Relation numérique entre les sexes ♀	♂
1	16	24.IX 2 29.IX 2 1.X 2 3.X 1 } 7 cm³	0	0	0
2	42	24.X 2 29.X 2 31.X 2 3.XI 0.5 } 6.5 cm³	1	0	1
3	15	28.IX 2 3.X 1 6.X 1 } 4 cm³	0	0	0
4	29	28.IX 2 3.X 1 6.X 1 } 4 cm³	16	0	16
5	92	16.IX 1 21.IX 1 23.IX 1 } 3 cm³	11	1	10
6	34	21.IX 1 26.IX 1 } 2 cm³	19	2	17
7	50	22.IX 1 29.IX 1 } 2 cm³	13	4	9
		La moyenne des expériences	60	11.06%	88.94%

Table 4

Immunisation des lapins avec des doses différentes de l'antisérum Y

Expériences №№	Lapins immunisés №№ des mâles	Date de l'immunisation et nombre des spermatozoïdes introdutis		Nombre des descendants	Relation numérique entre les sexes	
					♀	♂
1	63	24.X 1940	2	5	0	5
		29.X	2			
		2.XI	1			
			5 cm³			
2	16	28.X	1	2	0	2
		3.XI	1			
		6.XI	1			
		9.XI	1			
			4 cm³			
3	65	16.XI	1	18	7	11
		21.XI	1			
		25.XI	1			
			3 cm³			
4	48	2.XII	0.3	29	19	10
		7.XII	0.3			
			0.6 cm³			
5	15	24.XII	0.3	36	21	15
		29.XII	0.3			
			0.6 cm³			
6	9	3.I 1941	0.3	24	13	11
		8.I	0.3			
			0.6 cm³	89	59.55%	40.45%
7	55	3.II	0.5	28	21	7
		8.II	0.5			
			1 cm³	28	75%	15%

Les résultats de la table 1 indiquent que l'immunisation du mâle-producteur avec des spermatozoïdes X (anodiques) contribue au développement dans l'organisme de ce mâle de la spermatoxine anti-X, c'est-à-dire à l'affaiblissement des spermatozoïdes X, et à la prédominance du sexe masculin dans la postérité des mâles.

II série. Immunisation des mâles-producteurs avec des spermatozoïdes Y (cathodiques). En cas d'un dosage favorable la prédominance des femelles dans la descendance des mâles immunisés a été attendue.

Les résultats obtenus sont exposés dans la table 2.

Les résultats de la table 2 indiquent que l'immunisation avec des spermatozoïdes Y contribue à la prédominance du sexe féminin dans la descendance du mâle immunisé, mais le dosage doit être en ce cas minutieusement réglé. Le dosage le plus favorable, c'est l'introduction de 7000—9000 cellules.

III série. Immunisation des mâles-producteurs avec des antiséra (anti-X); la descendance attendue, prédominance du sexe masculin. Les résultats obtenus sont exposés dans la table 3.

Les résultats de la table 3 indiquent que la prédominance du sexe masculin dans la descendance a lieu non seulement après l'immunisation avec des spermatozoïdes X, mais aussi après l'immunisation avec l'antisérum X.

IV série. L'immunisation des mâles-producteurs avec l'antisérum Y; ce sont les femelles qui sont attendues dans la descendance du mâle immunisé. Les résultats des expériences sont exposés dans la table 4.

Les résultats des expériences déposés dans la table 4 indiquent que l'immunisation du mâle-producteur avec l'antisérum Y contribue à la prédominance des femelles dans la descendance du mâle immunisé.

Les résultats obtenus donnent une perspective d'application de cette méthode à la pratique de l'élevage.

Institut de Cytologie, d'Histologie et d'Embryologie.
Académie des Sciences de l'URSS.

Manuscrit reçu
le 24. XII. 1943.

LITTÉRATURE CITÉE

[1] В. Н. Шредер, Биологич. журн., I, № 5—6 (1933). [2] В. Н. Шредер, ibid., III, № 3 (1934). [3] В. Н. Шредер, ibid., V, № 4 (1936). [4] В. Н. Шредер, ibid., VI, № 5—6 (1937). [5] V. Schroeuder, Bull. Acad. Sci. URSS, sér. biol., № 3 (1940). [6] V. Schroeuder, ibid., № 2 (1941). [7] V. Schroeuder. C. R. Acad. Sci. URSS, XXVI, No. 7 (1940). [8] I. I. Metschnikoff, Ann. de l'Inst. Pasteur, XIV, No. 1, 378 (1900). [9] I. I. Metschnikoff, ibid., XIV, No. 6, 369 (1900). [10] I. I. Metschnikoff, ibid., XIII, No. 10 (1899). [11] H. Landsteiner, Zbl. f. Bakt. u Infektionskrankheiten, XXV, 546 (1897). [12] H. Landsteiner und van der Scheer, Proceed. Soc. Exp. Biol and Med., XXV, No. 1—3 (1927). [13] J. Cantacuzène, Ann. de l'Inst. Pasteur, XIV, No. 1, 378 (1900). [14] A. E. Teylor, Journ. Biol. Chem., V, No. 4 (1908). [15] M. F. Guyer, Journ. exper. Zool., 35, No. 2 (1922). [16] J. L. McCartney, Amer. Journ. of Physiology, LXVI (1923). [17] Н. Е. Костромина и П. Н. Карташев, Журн. микробиол. и инфекц. болезней, IV, 1 (1927). [18] W. Heule, G. Heule and L. Chambers, Journ. exp. Med., 68, 335 (1938). [19] S. Mudd and E. Mudd, Journ. of Immunology, XVIII, No. 1 (1929). [20] J. Jarcho, Amer. Journ. of Obstetrics and Gynecology, XVI, No. 4 (1928). [21] F. Kovacs, ibid., X, No. 4 (1925). [22] J. Baskin, M. D. Denver, F. Colo, ibid., XXIV, No. 4 (1932). [23] W. T. Pommerenke, Physiol. Zoology, I, No. 1 (1928). [24] G. Szantó, ZS. Ges. Exp. Med., 68, 335 (1938). [25] Z. Aszodi and G. Szantò, ibid., 97, H. 31 (1927). [26] R. Dittler, Münch. Med. Wochenschr., No. 52, 1425 (1920). [27] E. Waldstein u. R. Ekler, Wiener Klin. Wochenschr., No. 24 (1913). [28] H. Rosenfeld, Amer. Journ. of Obstetrics and Gynecology, XII, 385 (1926). [29] S. J. Fogelson, Surgery, Gynecology and Obstetrics, XLII, 374 (1926).

Prix $ 0.50